N4 Engineering Science

Gateways to Engineering Studies - Chris Brink

HYBRID LEARNING SOLUTIONS

Gateways to Engineering Studies

Engineering Science N4

Chris Brink

Published by
Hybrid Learning Solutions (Pty) Ltd

Email: urania@hybridlearning.co.za

ISBN: 978-1-928203-21-6

Editor: Urania Bellos
Proofreader: Urania Bellos
Book design: Sarah Buchanan
Cover design: Sarah Buchanan
Artwork: Wendi Wise / Sarah Buchanan

Printed and bound by: Formsxpress

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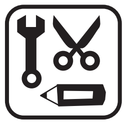
	Checklist		Practical
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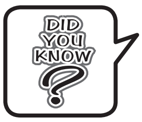
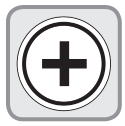
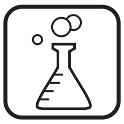
	Did you know?		Read
	Example		Safety
	Experiment		Site visit
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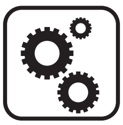
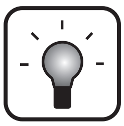
	In the workplace		Theoretical - questions, reports, case studies, etc.
	Keywords		Think about it

Notes

Module 1 Kinematics

Learning Outcomes

On the completion of this module the student must be able to:

- Solve problems dealing with linear motion analytically, where external factors such as wind and water are concerned. A maximum of tow simultaneous motions are examined.
- Solve analytical problems relating to practical situations where two objects move horizontally at constant velocity in different directions.
- Determine the results velocity, shortest distance, time, intersection, overtaking and actual velocity.
- Solve analytically problems that deal with projectiles which are launched vertically or at an angle other than 90º to the horizontal.

1.1 Introduction

Kinematics is a part of the science of mechanics is the study of pure motion which is irrespective of the force that causes it.

1.2 Resultant velocity

Resultant velocity is the sum of two or more velocities which have simultaneous influence on the object. It can be described as the effective velocity in a given direction. It is therefore the velocity of the object relative to earth or ground surface.
Resultant velocity occurs:

- When a passenger walks along the corridor of a moving train and has two simultaneous velocities.
- When a sailor walks across the deck of a yacht sailing over a flowing river.
- When a plane flies in a moving air mass or wind.

Definition: Resultant velocity
Resultant velocity simply refers to the sum of all vectors. It is calculated by adding two velocities in the same direction using the Pythagorean theory. A good example would be a plane flying amidst a tailwind. When you add the velocities of the tailwind and the plane, what you get is known as resultant velocity.

Worked Example 1.1

A plane flies at a velocity of 670 km/h from the North East, direction according to its compass. The wind is blowing at a velocity of 110 km/h from West to East. Calculate the resultant velocity of the plane in magnitude and direction.

Solution:

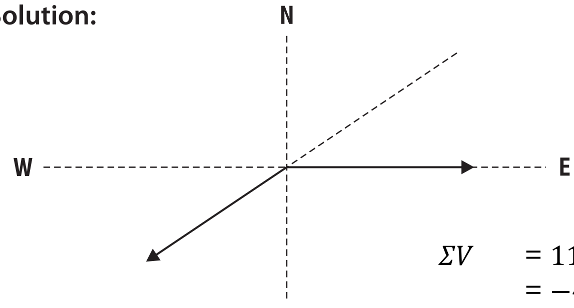


Figure 1.1

$$\Sigma V = 110 \text{ Sin } 0° + 670 \sin 225°$$
$$= -473{,}762\ km/h$$
$$\Sigma H = 110 \cos 0° + 670 \cos 225°$$
$$= -363{,}762$$

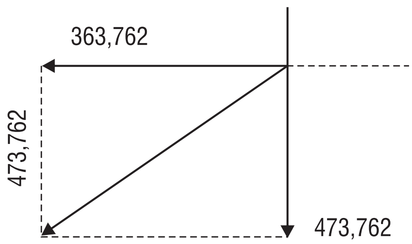


Figure 1.2

$$R = \sqrt{(363{,}762)^2 + (473{,}762)^2}$$
$$= 597{,}305\ km/h$$
$$\theta = \tan^{-1} \frac{473{,}762}{363{,}762}$$
$$= 52{,}424°$$
$$R = 597{,}305 km/h\ W\ 52{,}424° S$$

OR

$$R = 597{,}305 km/h\ S\ 37{,}516° W$$

An alternative method

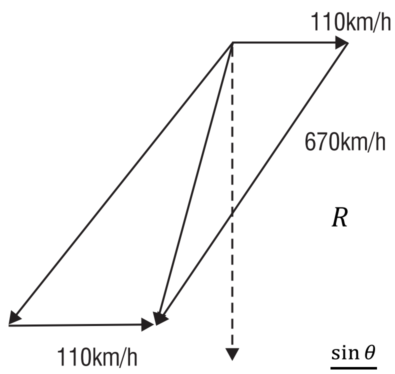


Figure 1.3

$$R = \sqrt{(110)^2 + (670)^2 - 2(110)(670)\cos 45°}$$
$$= 597{,}304\ km/h$$
$$\frac{\sin\theta}{110°} = \frac{\sin 45}{597{,}304}$$
$$\sin\theta = \frac{110 \times \sin 45°}{597{,}304}$$
$$\sin\theta = 0{,}1302$$
$$k = 481°$$
$$\alpha = 37{,}519°$$
$$R = 597{,}304\ S\ 52{,}424° W$$

1.2 Relative velocity

If two bodies A and B are moving in a straight line in the same direction with velocities u and v respectively, where $u > v$, the displacement between the two bodies increase at the rate of $u - v$. This is the velocity of A relative to the velocity of B.

If B is moving in the opposite direction to A, the displacement between them increases at the rate $u + v$. This is again the velocity of A relative to the velocity of B. In each case the relative velocity of A with respect to B is the sum of A's velocity and B's velocity reversed. To find the relative velocity it is not necessary to know the positions of the bodies, but only the magnitudes and directions of their velocities. The relative velocity of B with respect to A is equal and opposite to the relative velocity of A with respect to B.

Worked Example 1.2

A submarine travels in a direction North 30° East and fires a torpedo in a northerly direction. The submarine travels at 30 km/h and the speed of the torpedo is 80 km/h. Calculate the velocity of the torpedo relative to the velocity of the submarine in magnitude and direction.

Solution:

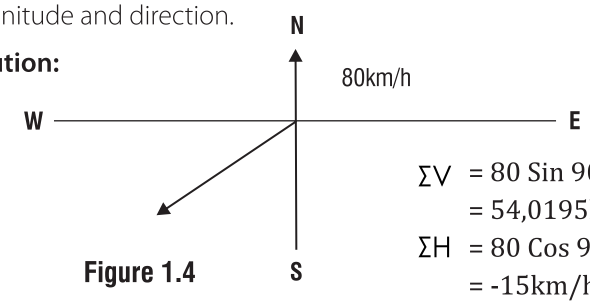


Figure 1.4

$\Sigma V = 80 \text{ Sin } 90° + 30 \text{ Sin } 240°$

$= 54{,}0195 \text{km/h}$

$\Sigma H = 80 \text{ Cos } 90° + 30 \text{ Cos } 240°$

$= -15 \text{km/h}$

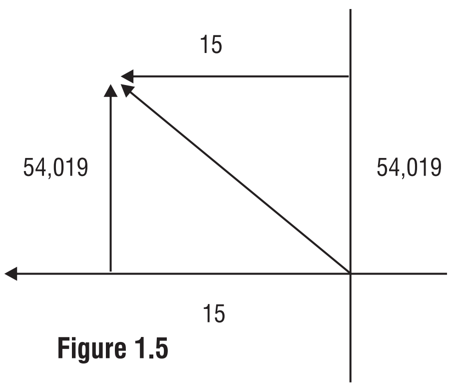


Figure 1.5

$R = \sqrt{(54{,}019)^2 + (15)^2}$

$= 56{,}063 \text{ km/h}$

$\theta = \tan^{-1} \frac{54{,}019}{15}$

$= 74{,}481°$

$V_s = 56{,}063 \text{ km/h W } 74{,}481° \text{ N}$

OR

$V_s = 56{,}063 \text{ km/h W } 74{,}481° \text{ N}$

An alternative method

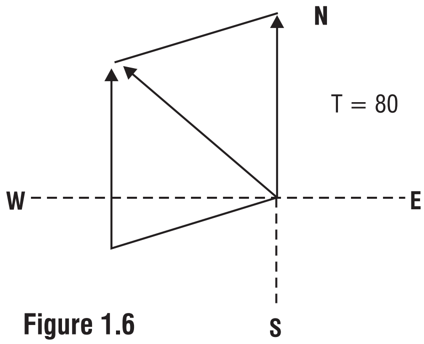


Figure 1.6

$R = \sqrt{(80)^2 + (30)^2 - 2(80)(30)\cos 30°}$

$= 56{,}063 \text{ km/h W}$

$\frac{sin\theta}{30} = \frac{\sin 30°}{56{,}063}$

$\text{Sin } \theta = \frac{30 \times \sin 30°}{56{,}063}$

$\theta = \sin^{-1}(0{,}2677)$

$= 15{,}527°$

${}_TV_S = 56{,}063 \text{ km/h N}15{,}527\text{W}$

OR

${}_TV_S = 56{,}063 \text{ km/h W}74{,}473 \text{ N}$

1.3.1 Relative path

Let the positions of the two bodies A and B, in **Figure 1.7**, at some point be C and D respectively. If the relative velocity of A with respect to B is known, the relative path of A with respect to B is straight line through C in the direction of the relative velocity.

Any queries relating to the nearest approach of the two bodies may be handled by assuming B to be stationary at D and A moving along the relative path with the relative velocity.

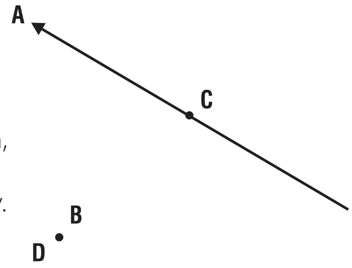


Figure 1.7

The shortest distance apart of the two bodies is the perpendicular distance from D to the relative path.

Worked Example 1.3

Two ships leave Port Elizabeth harbor simultaneously. Ship Q travels south at 104km/h and ship P travels south-west at 127km/h.

Calculate the velocity of ship Q relative to the velocity of Ship P in magnitude and direction.

Solution:

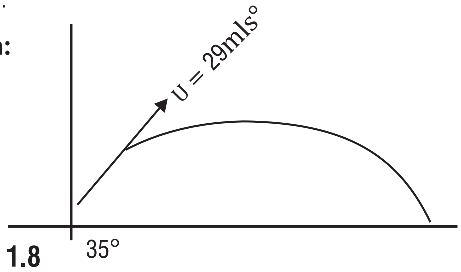


Figure 1.8

From the first principle

$$S = \frac{U^2 \sin^2\theta}{2g} = \frac{29^2 \sin^2 35}{2 \times 9{,}8} = 14{,}116\text{m}$$

An alternative method

$$S = \frac{V^2 - U^2}{2 \times g} = \frac{0^2 - (29 \sin 35)^2}{2 \times -9{,}8} = 14{,}116\text{m}$$

or

$$T_{vert} = \frac{v-u}{g} = \frac{0 - 29 \sin 35}{-9{,}8} = 1{,}697\text{ s}$$

$$S = U t + \tfrac{1}{2} g t^2$$
$$= 29 \sin 35 \times 1{,}697 + 0{,}5 \times -9{,}8 \times 1{,}697^2$$
$$= 14{,}116\text{ m}$$

From first principles

$$S_{hor} = \frac{U^2 \sin 2\theta}{g} = \frac{29^2 \sin 2 \times 35°}{9{,}8} = 80{,}641\text{ m}$$

An alternative method

$$S_{hor} = U_{hor} \times t_{hor}$$
$$= 29 \cos 35 \times (2 \times 1{,}697)$$
$$= 80{,}626\text{ m}$$

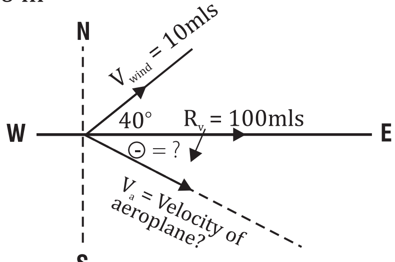


Figure 1.9

$$R_{velocity} = \frac{S}{T} = \frac{300\,000}{50 \times 60} = 100\text{m/s}$$

$$\Sigma\text{Vertical} = 10 \sin 45° + V_a \sin\theta$$
$$0 = 7{,}07 + V_A \sin\theta$$
$$V \sin\theta = -7{,}07\text{ m/s}$$

1.4 Projectiles

1.4.1 Vertical movement

Note: Important formulae

- $S = \frac{u+v}{2} \times 1$
- $S = ut + \frac{1}{2}gr^2$
- $v^2 = u^2 + 2gS$
- $g = \frac{v-u}{t}$

Worked Example 1.4

A stone is thrown at a velocity of 42 m/s at an angle of 26° to the horizontal.
Calculate the following:

a) The maximum height that the stone reaches.
b) The horizontal displacement of the stone.

Solution:

From the first principle

a) S(vertical) = $\frac{U^2 sin^2\theta}{2g}$

$= \frac{42^2 sin^2 26^\circ}{2 \times 98}$

$= 17{,}295\text{m}$

b) S(horizontal) = $\frac{U^2 sin2\theta}{g}$

$= \frac{42^2 sin2 \times 26^\circ}{9{,}8}$

$= 141{,}841\text{m}$

An alternative method

Equations of motion

c) S(max) $= \frac{V^2 - U^2}{2g}$

$= \frac{0^2 - (415m26)}{2\, x - 9{,}8}$

$= 17{,}295\text{m}$

d) S(Range) $= U\ cos\theta \times \frac{2(U\ sin\theta)}{g}$

$= \frac{42^2 cos26^\circ \times 2 \times 42\ sin42}{9{,}8}$

$= 141{,}841\text{m}$

4.2.2 Parabolic motion

Figure 1.10 shows an objected projected with an initial velocity of u in a direction making an angle α with the horizontal OX. The motion is in the vertical plane YOX.
The only acceleration of the object is due to gravity and is $-g$ parallel to the y-$axis$.
The initial velocities of the object in the x and y directions are $u \cos \alpha$ and $u \sin \alpha$ respectively.

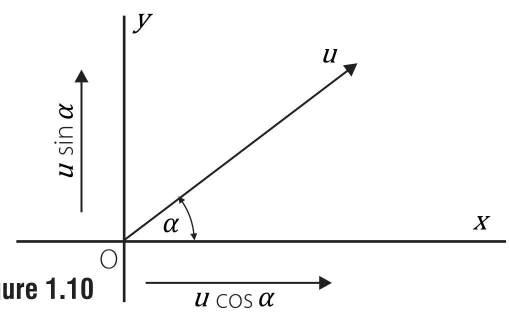


Figure 1.10

The horizontal component of the velocity $u \cos \alpha$, remains constant through the motion. The vertical component which is $u \sin \alpha$ decreases to zero as time increases and ultimately becomes negative.

Vertical displacement: $y = u(\sin\alpha)t - \frac{1}{2}gt^2$ ....(1)

Horizontal displacement: $x = (u\cos\alpha)t$ ....(2)

From (2): $t = \dfrac{x}{u\cos\alpha}$

Substitute t in (1): $y = (u\sin\alpha)\dfrac{x}{u\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2$

$= x\tan\alpha - \dfrac{g \times x^2}{2u^2\, cos^2\,\alpha}$

This equation is the equation of a parabola, of the second degree in x only. The time for the horizontal displacement or range, is twice the time to reach the maximum height. For a given velocity of projection the range varies with the angle of projection α. The range has it maximum value when α is 45°.

Worked Example 1.5

An object is projected at an angle such that the horizontal displacement (range) is three times more than the maximum vertical height.
Calculate the angle of projection.

Solution:

From the first principle: $S_H = 3 \times S_V$

$\dfrac{U^2 \sin 2\theta}{g} = \dfrac{U^2 \sin^2\theta}{2g} \times 3$

$\sin 2\theta = \sin^2\theta \times \frac{3}{2}$

$\dfrac{2}{3} = \dfrac{\sin\theta \times \sin\theta}{2Sin\,\theta\; Cos\,\theta}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{4}{3}$

$\tan\theta = 1{,}333$

$\theta = 53{,}13°$

Us may also use equations of motion

Activity 1.1

1. A man walks to the back of a train at a speed of 2 m/s while the train is moving forward at 12 m/s. Calculate the man's rate of velocity relative to the ground [10m/s]
2. A ship sails at 20 m/s^{-1} due north and experiences a current of 5 m/s^{-1}.
   Calculate the magnitude and direction of the resultant velocity when the current is flowing:
   (a) west [20,62 m/s^{-1} 14,03° west of north]
   (b) east [20,62 m/s^{-1} 14,03° east of north]
3. A cyclist travels at 15 m/s^{-1} due west. Calculate the velocity and direction of the wind as the cyclist encounters it when:
   (a) a wind is blowing at 8 m/s^{-1} from the north [17 m/s^{-1}, 28,07° south of east]
   (b) a wind is blowing at 6 m/s^{-1} from the south [16,16 m/s^{-1}, 21,8° north of east]
   (c) a wind is blowing at 5 m/s^{-1} from the east [10 m/s^{-1} to the west]
   (d) a wind is blowing at 6 m/s^{-1} from the west [21 m/s^{-1} to the east]
4. A boat crosses a river upstream at an angle of 135° against the direction of the stream. Due to the common effect of the current and the velocity of the boat, the boat crosses the river at a right angle and at 10 m/s^{-1}.
   Calculate:
   (a) the velocity of the stream [10 m/s^{-1}]
   (b) the velocity of the boat in still water [14,14 m/s^{-1}]
5. A velocity of 30 km/h west and a velocity of 40 km/h north are added vectorially to obtain the resultant velocity of a ship with respect to the land.
   Calculate the magnitude and direction of the resultant velocity. [50 km; 53,13° north of west]
6. A plane can fly at 90 m/s-1 in calm conditions. A wind is blowing due north. At 30m/s-1. Calculate the relative velocity of the plane with respect to the ground when it is flying in a
   (a) northerly [120 m/s^{-1}]
   (b) easterly [94,9 m/s^{-1} in a direction 71,6° E of N]
   (c) southerly direction [60 m/s^{-1}]
7. (a) What is meant by negative velocity?
   (b) Explain what is meant by the term reference framework.
   (c) Explain the term relative velocity.
   (d) A motorcar is moving at a constant velocity of 20 m/s^{-1} in a straight line.
   Calculate the two possible velocities of a second motorcar for it to obtain a relative velocity of 25 m/s-1. With respect to the first motorcar. [45 m/s^{-1} ; 5 m/s^{-1}]
8. A river is flowing north at 2 m/s^{-1}. A motorboat has a maximum velocity of 4m/s^{-1} in still water. Calculate the relative velocity (resultant velocity) of the boat with respect to the river-bank if:
   (a) it is moving downstream [6 m/s^{-1} north]
   (b) it is moving upstream. [2 m/s^{-1} south]
9. A motorboat with a velocity 10 m/s^{-1} with respect to the water, is moving downstream at an angle of 600 relative to the flow of the water. The water is flowing at 4 m/s^{-1}. Calculate the resulting velocity of the boat. [12,5 m/s^{-1} at an angle of 44° relative to the flow of the water.]

Activity 1.2

1. A plane is flying horizontally at a height of 1 000 m, at 500 km/h and releases a bomb.
   Calculate:
   (a) the time taken for the bomb to reach the ground [14,14 s]
   (b) the velocity of the bomb when it reaches the ground [141,4 m/s^{-1}]
   (c) the horizontal displacement of the bomb from the time it leaves the plane until it hits the ground. [1,96 km]
2. A body is accelerated horizontally and uniformly from rest at 0,8 m/s^{-2},
   Calculate:
   (a) the velocity after 10 s [8 m/s^{-1}]
   (b) the distance after 10 s [40 m]
   (c) the distance travelled during the eighth second [6 m]
3. A man on a bridge drops a stone and it takes 3 s to reach the water in the river.
   Calculate:
   (a) the velocity with which it hits the water [30 m/s^{-1})
   (b) the distance between the man and the water. [45m]
4. A bullet is fired at an angle of 30° to the horizontal at a velocity of 350 m/s^{-1}.
   Calculate:
   (a) the maximum height it will reach [1531,25m]
   (b) the time taken to reach its maximum height [17,55 s]
   (c) the displacement of the bullet when it hits the ground. [10,61 km]
5. A car must cover a distance of 200 km during two hours. The first 60 km is covered at an average speed of 80 km/h. Calculate the average speed for the rest of the journey. [112 km/h]
6. A motorcar accelerates, uniformly, from 8 m/s^{-1} on a horizontal road and covers a distance of 200 m during the first 15s.
   Calculate: .
   (a) the deceleration [0,71 m/s^{-2}]
   (b) the velocity after 15 s [18,67 m/s^{-1}]
   (c) the average-velocity [13,33 m/s-1]
7. Name all the symbols in $ut + \frac{1}{2}at^2$ and write down the SI units of the quantities.
8. An object moves from A due west to B from rest and at uniform acceleration of 0,8m/s^{-2}, during 10 s. It then moves to C, due north, at a constant velocity for 400m. It is then uniformly decelerated so that it moves at 0,5 m/s^{-1} when it passes a point D800 m from C.
   Calculate:
   (a) the distance between A and B [40 m]
   (b) the velocity when it reaches point B [8 m/s^{-1})
   (c) the time to move from B to C [50s]
   (d) the deceleration between C and D [0,04 m/s^{-2}]
   (e) the time to cover the distance between C and D [187,5 s]
   (f) the total displacement. [1 200,67 m]
9. A car is moving at a constant velocity of 80 km/h on a horizontal road and is brought to rest over a distance of 100 m.
   Calculate:
(a) the deceleration. [2,469 m/s-2]
(b) the time taken by the car to come to rest. [9 s]

continued overleaf ...

10. A stone is thrown vertically upwards from a building that is 20 m high. It takes 4 s to reach its maximum height and falls freely, just next to the building.
    Calculate:
    (a) The maximum height (from the ground) that it will reach [100 m)
    (b) the velocity of the stone when it passes the top of the building [40 m/s^{-1}]
    (c) the velocity just before it reaches the ground [44,72 m/s^{-1}]
    (d) the total time taken to reach the ground [8,472 s]
11. A stone is thrown vertically upwards and reaches the ground after 5 s.
    Calculate:
    (a) the maximum velocity [25 m.s^{-1}]
    (b) the maximum height [31 125m]

Self Check

I am able to:	YES	NO
• Solve problems dealing with linear motion analytically, where external factors such as wind and water are concerned. A maximum of two simultaneous motions are examined.	○	○
• Solve analytical problems relating to practical situations where two objects move horizontally at constant velocity in different directions.	○	○
• Determine the results velocity, shortest distance, time, intersection, overtaking and actual velocity.	○	○
• Solve analytically problems that deal with projectiles which are launched vertically or at an angle other than 90º to the horizontal.	○	○

If you have answered "no" to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Notes

Module 2 Angular Motion

Learning Outcomes

On the completion of this module the student must be able to:

- Calculate angular displacement, velocity and acceleration.
- Give the relationship between linear and angular motion and apply this relationship to calculations which deal with displacement, velocity and acceleration.
- Calculate the torque required for acceleration or for braking. Resistance to motion and friction can be included.
- Calculate the work done and the power required as in 3 and also calculate efficiency of transmission if input power is given.

2.1 Introduction

Angular motion usually refers to the rotating motion of objects, for example a flywheel, a motor, a pendulum or the spin-drier of a washing machine.

2.2 Angular displacement, angular velocity and angular acceleration

An angle is measured in degrees or radians, for example angle θ in **Figure 2.1** is subtended by arc AA_1.

One radian is the angle subtended at the centre of the circle by an arc (or arcs) equal in length to the radius. $360° = 211\pi$ radians.

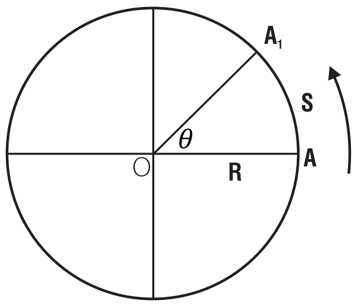


Figure 2.1

From **Figure 2.1** it follows that:

Angular displacement is the angle formed, in radians, during motion. It is 2π radians when radius (OA) of **Figure 2.1** makes 1 revolution. It could be any value eg $3(2\pi)$ or $3(2\pi)$+1,6 etc.

If radius OA rotates at a constant speed the rate of change by which θ is formed is also constant. This rate of change of the angle θ is known as angular velocity (ω). $\omega = \frac{\Delta\theta}{t}$ (θ in radians)

If radius OA completes 1 revolution in 1 s, the angular velocity $\omega = 2\pi\ rad/s$. If radius OA completes 2 revolutions in 1 s, $\omega = 2\pi(2) rad/s$. In the general case $\omega = 2\pi n\ rad/s$ where n, the rotational frequency, is given in r/s.

If the radius OA does not rotate at a constant speed but accelerates uniformly, the rate of change of the angular velocity is called angular acceleration (α).

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi n_2 - 2\pi n_1}{t} = \frac{2\pi(n_2 - n_1)}{t}.$$

2.3 Circumferential velocity

If point P on the circumference of the circle in **Figure 2.2** makes one revolution in 1 s, the distance covered is equal to the circumference of the circle, πD. The circumferential velocity will be πD(1) units/s.

If 2 revolutions are completed in 1 s, the circumferential velocity (v) will be πD(2). In the case where n revolutions are completed in 1 s it follows the $v = \pi Dn$, where v = circumferential velocity, D = diameter and n = rotational frequency in r/s.

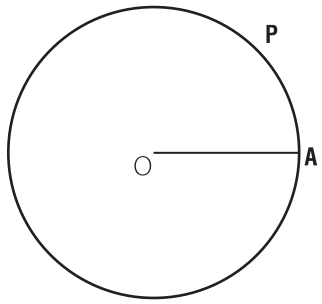


Figure 2.2

2.4 The relation between angular, velocity, angular acceleration, linear velocity and linear acceleration

$$v = \pi Dn$$
$$= \pi(2R)n$$
$$= (2\pi n)R$$
$$= \omega R \text{ because}$$
$$\omega = 2\pi n$$

v = linear velocity
D = diameter
n = rotational frequency
ω = angular velocity

$$\alpha = \frac{v_1 - v_2}{t}$$
$$= \frac{\omega_2 R - \omega_1 R}{t}$$
$$= \frac{\omega_2 - \omega_1}{t}.R$$
$$= \alpha R (\alpha \ in \ rad.s^{-2})$$

Summary of formulae

- $\omega = 2\pi n$
- $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$
- $v = \omega R$
- $\alpha = \alpha R$
- $s = \theta R$

Worked Example 2.1

A flywheel rotates at 8 r/s. If the radius of the wheel is 2m, calculate:

(a) the angular velocity of the wheel at any moment
(b) the linear velocity of a point on the circumference of the wheel at any moment
(c) the angular displacement during 0,2 s

Solution:

Given: n = 8 r/s, R = 2 m

(a) $\omega = 2\pi n$
$= 2\pi n(8)$
$= 50{,}27 \text{ rad.s}^{-1}$

(b) $v = \omega R$
$= 50{,}27(2)$
$= 100{,}5 \text{ m.s}^{-1}$

(c) In 1 s, θ = 50,27 rad
In 0,2 s, θ = 50,27 x 0,2
= 10,05 rad

Worked Example 2.2

A point on the circumference of a wheel with a diameter of 6 m moves at a linear speed of 8 m.s^{-1}. Calculate the angular velocity at any moment.

Solution:

Given: $D = 6\ m \ \therefore R = 3m, v = 8\ m.s^{-1}$

$$v = \omega R$$
$$8 = \omega(3)$$
$$\omega = \frac{8}{3}$$
$$= 2{,}67\ rad.s^{-1}$$

Worked Example 2.3

The spin drier of a washing machine accelerates uniformly from 10r/min to 800r / min during 4 s.
Calculate:
(a) the angular acceleration
(b) the linear acceleration of a point 0,2 m from the centre
(c) the average angular velocity
(d) the angular displacement during 4 s
(e) the distance travelled by a point 0,2 m from the centre
(f) the number of revolutions.

Solution:
Given: $n_1 = 10r/min, n_2 = 800r/min \text{ and } t = 4s$

(a) $n_1 = 10$ r/min
$= \frac{10}{60}$
$= 0,167$ r/s
$\alpha = \frac{2\pi(n_2 - n_1)}{t}$
$= \frac{2\pi(13,11 - 0,167)}{4}$
$= 20,68$ rad.s^{-2}

$= 800$ r/min
$= \frac{800}{60}$
$= 13$ 33 r/s

(b) $a = \alpha R$
$= 20,68 \times 0,2$
$= 4,14$ m.s^{-2}

(c) $\omega_1 = 2\pi n_1$
$= \pi(0,167)$
$= 1,05$ rad,s^{-1}

$\omega_2 = 2\pi n_1$
$= 2\pi(13,33)$
$= 83,75$ rad.s^{-1}

Average angular velocity
$= \frac{\omega_1 + \omega_2}{2}$
$= \frac{1,05 + 83,75}{2}$
$= 42,4$ rad.s^{-1}

(d) $\theta = \frac{\omega_1 + \omega_2}{2} . t$
$= 42,4 \times 4$
$= 169,6$ rad

(e) $s = \theta R$
$= 169,6(0,2)$
$= 33,92$ m

(f) n average $= \frac{(n_1 + n_2)}{2}$
$= \frac{0,167 + 13,33}{2}$
$= 6,75$ r/s

During4s ,n $= 6,75 \times 4$
$= 27$ r

Or $n = \frac{s}{circumference}$
$= \frac{33,92}{0,4\pi}$
$= 27$ r

2.5 Work done by torque

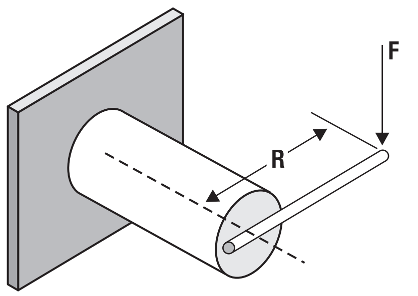


Figure 2.3

The moment of a force about an axis is the energy transferred to the axis by the force due to its magnitude and position. It is calculated as the product of the force and the perpendicular distance between the working line of the force and the turning point. When a shaft is subjected to a. torque, the material of the shaft resists the torque.

Torque (T) = force (F) x perpendicular distance between turning point and working line of the force (R)

T = F x R

For one revolution:

Work done = force x distance
= force x πD
= force x π x 2R (2R = D)
= 2π(force x R)
= 2πT

Power = $\frac{work\ done}{time}$ = 2πnT W

where n = r/s and T = N.m

where T = torque, which is equal to load x R

Work done during **n** revolutions = n(2πT)
= 2πnT J

2.5.1 Work done if the force describes an arc

Work done = F x arc
= F x Rθ (arc length = R xθ,
= (F x R) θ where θ is in radians)
= Tθ

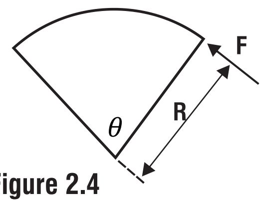


Figure 2.4

Worked Example 2.4

The torque applied to a nut by a spanner with a length of 0,2 m is 48N.m.
Calculate the magnitude of the force applied to the end of the spanner.

Solution:

R = 0, 2m; T = 48 N.m; F = ?
T = F x R
48 = F x 0,2
F = 240 N

Worked Example 2.5

A force of 200 N is applied to the end of a spanner. The perpendicular distance between the nut and the working line for the force is 0,5m.
Calculate:
(a) the torque
(b) the work when the nut is turned through an angle of 30° while the torque remains constant.

Solution:

Given: F = 200 N; R = 0,5 m

(a) T = F x R
= 200 x 0,5
= 100 N.m

(b) 360° = 2π radians
30° = x radians
$\therefore \frac{x}{30} = \frac{2\pi}{360}$
$x = \frac{2\pi}{360} \times 30$
= 0,524 radians

OR

180° = π radians
1° = $\frac{\pi}{180}$

30° = $\frac{\pi}{180} \times 30$

= 0,524 radians

Work done = T x θ
= 100 x 0,524
= 52,4 J

An alternative method

Worked Example 2.6

A machine has a torque of 15 N.m at its spindle. The diameter of the spindle is 0,4m and its rotational frequency is 2 rad/s.

Calculate:

(a) the magnitude of the load on the rope around the spindle

(b) the work done by the machine during 30 s

(c) the output power.

Solution:

Given: T = 15 N.m; D = 0,4 m; n = 2 r/s

(a) $T = W \times R$

$15 = W \times 0{,}2$

$W = 75N$

(b) Work done per revolution $= 2\pi T$

$= 2\pi(15)\ J$

In one second the spindle rotates through 2 radians. In 30 seconds the spindle rotates through 60 radians.

60 radians $= \frac{60}{2\pi}$ revolutions

Work done during 30 s $= 2\pi(15) \times \frac{60}{2\pi}$

$= 900\ J$

(c) Power $= 2\pi nT$ (where n = r/s)

$= 2\pi \times \frac{2}{2\pi} \times 15$

$= 30\ W$

Activity 2.1

1. A flywheel rotates at 15 ra^{-1}.
Calculate:
(a) the angular velocity at any moment [94,2 $rad.s^{-1}$]
(b) the circumferential speed at a point 0,8 m from the centre. [75,4 $m.s^{-1}$]
2. An axle rotates at 8 $rad.s^{-1}$. Calculate the rotational frequency in r/s and r/min as well as the angular displacement during 5 s. [1,27 $r.s^{-1}$; 76,4 r/min; 6,35 rad]
3. The angular displacement of a wheel with a diameter of 0,6 m, is 1,5π during 1,2 s. If the wheel rotates at a constant angular speed calculate:
(a) the distance covered by a point on the circumference of the wheel [1,41m]
(b) the angular velocity at any moment [3,9 $rad.s^{-1}$]
(c) the circumferential speed.(1,17 $m.s^{-1}$]
4. A wheel rotates at a constant speed of 600 r/min. The diameter of the wheel is 0,4m.
Calculate:
(a) the angular velocity at any moment. (62,8 $rad.s^{-1}$]
(b) the circumferential velocity of a point on the circumference of the wheel at any moment (12,6 $m.s^{-1}$]
(c) the angular displacement during 2 s. [125,6 rad]
(d) the distance covered by a point on the circumference of the wheel during 2 s. [25,2m]
5. A car rounds a curve with a radius of 12 m at 60 km/h. Calculate:
(a) the angular velocity of the car at any moment [1,39 $rad.s^{-1}$]
(b) the angular displacement if the length of the curve is 20m. [1,67 rad]
6. An axle rotates at 8 $ra.d.s^{-1}$ and is retarded to 2 rad.s-1 during 5 s. Calculate the angular deceleration. (1 ,2 $rad.s^{-2}$]
7. The rotational frequency of an axle is increased uniformly from rest to 600r/min during 6 s.

Calculate:
(a) the average rotational frequency in r/s (5 r/s]
(b) the angular displacement during 3 s [30π rad]
(c) the angular acceleration in $rad.s^{-2}$ [10,47 $rad.s^{-2}$]

8. Calculate the number of revolutions that a rotating disc will make if it is uniformly accelerated from rest to 1 $rad.s^{-1}$ during 8 s. Also calculate the angular acceleration and the linear acceleration of a point 0,6 m from the centre. [4,5 r; 0,875 $rad.s^{-2}$; 0,53 $m.s^{-2}$]
9. An emery wheel with a diameter of 0 4 m takes 4 s to reach a required circumferential velocity of 1 $m.s^{-1}$ if it is accelerated uniformly from rest.
Calculate:
(a) the average angular velocity. (17,5 $rad.s^{-1}$]
(b) the angular acceleration (8,75 $rad.8^{-2}$]
(c) the linear acceleration of a point on the circumference of the wheel [1,75 $m.s^{-2}$]
(d) the angular displacement during 4 s. (70 rad]
(e) the distance covered by a point on the circumference of the wheel during the 4 s. [14m]
10. A stirrer rotates at 800 r/min and accelerates to 2 000 r/min during 60 revolutions.
Calculate:
(a) the time required for the acceleration [2,57s]
(b) the angular acceleration. (48,9 rad.s-2]

Activity 2.2

1. A force acting on a rotating arm of length 1,2 m varies from 0 N to 200 N during one revolution. Calculate the average torque and the work done during one revolution. [120 N.m; 754 J]

2. A load of 1 900 N is lifted by the drum of a lifting device. The diameter of the drum is 0,2 mm. Calculate the power if the drum rotates at 14 r/min.
[278,6 WJ]

3. A load of 900 N is suspended vertically from a hoist drum of 500mm diameter. Calculate the magnitude of the torque exerted on the shaft of the drum. [225 N.m]

4. The torque on the spindle pulley of a machine is 50 N.m. Calculate the cutting force on both sides of a drill of diameter 14 mm fitted to the spindle. [3,57 kN]

5. A mass of 8 kg rests on a horizontal plane. A horizontal force of 40 N is applied to the mass to displace it by 50 m. The resistance against motion is 15 N.
Calculate:
(a) the work done by the resistance force [750 J]
(b) the work converted into kinetic energy [1 250 J]
(c) the total power when the force is applied for 10 s. [2 kJ; 200 W]

6. An electric motor has a pulley with a diameter of 250 mm and drives a drilling machine with the aid of a rope. The diameter of the pulley on the drilling machine is 120 mm. The torque on the rope pulley of the motor is 50 N.m.
Calculate:
(a) the effective tension in the rope [400N]
(b) the torque on the pulley of the drilling machine [24 N]
(c) the power transferred to the drill if the pulley of the drill rotates at 15 r/s. [4,71kW]

7. The tractive force in both sides of a drive belt is 400 N and 200 N respectively. The diameter of the drive pulley is 0,8 m and it rotates at 150r/min.
Calculate:
(a) the torque on both sides [160 N.m; 80 N.m]
(b) the difference in torque (it is the effective torque) [80 N.m]
(c) the power transmitted by the forces separately [2,51 kW; 1,26 kWJ]
(d) the power transmitted by the driving pulley. [1,25 kWJ]

8. The torque on a gear driven directly by a machine is 120 N.m. The power transmitted by the gear is 4 kW. Calculate the rotational frequency of the machine. [5,3 rls]

Self Check

I am able to:	YES	NO
• Calculate angular displacement, velocity and acceleration.	○	○
• Give the relationship between linear and angular motion and apply this relationship to calculations which deal with displacement, velocity and acceleration.	○	○
• Calculate the torque required for acceleration or for braking. Resistance to motion and friction can be included.	○	○
• Calculate the work done and the power required as in 3 and also calculate efficiency of transmission if input power is given.	○	○

If you have answered "no" to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Module 3 Dynamics

Learning Outcomes

On the completion of this module the student must be able to:

- State Newton's three laws of motion and calculate the total force required for motion on horizontal and inclined planes with regards to practical problems, including gravitational force, frictional force and inertia.
- Calculate the work done over a given distance or during a given time and calculate the power at a given instant or at a given velocity.
- Define the concepts kinetic and potential energy and state the law of conservation of energy.
- Apply the law of conservation of energy to a free-falling body and to motion on an inclined plane where no external force is applied.

3.1 Introduction

Dynamics is the science of the forces that cause motion. It entails studying the forces that either change or tend to change, the uniform motion in a straight line of a body or the state of rest of a body.

Did you know?
Dynamics is the science of the forces that cause motion.

3.2 Newton's laws

Sir Isaac Newton formulated his famous laws in 1686.

3.2.1 Newton's first law

A body keeps its position at rest or its uniform motion in a straight line unless external forces change this state. Newton's first law is also known as the law of inertia. The property of a body to remain in a state of rest or uniform motion is called its inertia. A body is inert because of its mass; the greater the mass, the greater the inertia.

When a man is travelling by car and the brakes are applied, the car will slow down but the man will tend to keep his forward motion.

3.2.2 Newton's second law

The change of momentum per time unit is proportional to the applied force and takes place in the direction of the working line of the force.

Definition: Newton
The newton is the unit of force in the SI system. It is defined as the force required to give a mass of 1kg an acceleration of 1 m.s $^{-2}$.

Momentum $= m.v$
Change in momentum $= mv_2 - mv_1$
It follows from Newton's law that:

$$F \propto \frac{mv_2 - mv_1}{t}$$

$$F \propto \frac{m(v_2 - v_1)}{t}$$

$$F \propto ma$$

$$F = ma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Substitute $F = 1\,N;\ m = 1\,kg;$
$a = 1\,m.s^{-2}$ into (1)
$\therefore 1 = k(1)(1)$
$\therefore k = 1$
$\therefore$ (1) becomes $F = ma$ ....(2)
When a body is moving only under the influence of gravity, (2) becomes $F = mg$, where F is the weight due to gravitational force.

3.2.3 Newton's third law

For every action there is an equal and opposite reaction.

Note:
The force F in (2) is the resultant force in the direction of the acceleration.

Summary of formulae

Only one formula is important for this syllabus: $F = mg$

Worked Example 3.1

A car with a mass of 0,8 x 103 kg, is moving at 80 km/h on a horizontal plane and is braked to rest over a distance of 50m.
Calculate:
(a) the deceleration
(b) the braking force.

Solution:

u = 80 km/h = 22,22 $m.s^{-1}$; v = 0; s =50 m
m = 0, 8 x 10^3 kg

(a) $a = \frac{v^2-u^2}{2s}$
$= \frac{0-22,22^2}{2(50)}$
$= -4,94\ m.s^{-2}$
$\therefore$ The deceleration is $4,94\ m.s^{-2}$

(b) $F = ma$
$= 0,8 \times 10^3(-4,94)$
$= -3,962 \times 10^3$ N
The negative sign indicates that it is a braking force.

Worked Example 3.2

A mass of 80 kg accelerates uniformly from 3$m.s^{-1}$ to 12 $m.s^{-1}$ in 20s, in a straight line up an inclined plane of 1:20. The resistance against motion is 300N. Calculate the force causing the acceleration.

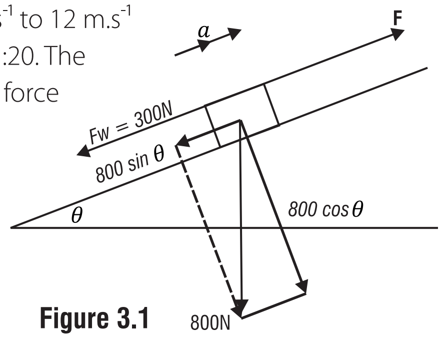


Figure 3.1

Solution:

It is necessary to calculate the acceleration first.

$a = \frac{v-u}{t}$
$= \frac{12-3}{20}$
$= \frac{9}{20}$
$= 0,45\ m.s^{-2}$

The acceleration is upwards, therefore F is greater than the sum of the downward forces.

$\therefore$ The accelerating force $= F - (300 + 800 \sin\theta)$

$Tan\,\theta = \frac{1}{20}$
$\theta = \tan^{-1}\frac{1}{20}$
$\theta = 2,86°$

$\therefore$ From $F = ma$:
$F - 300 - 800 \sin 2,86° = ma$
$F - 300 - 39,92 = 80(0,45)$
$F = 339,92 + 36$
$= 375,92\ N$

Worked Example 3.3

Calculate the force F required to accelerate the 9 kg mass uniformly at 0,5m.s^{-2}. The frictional force acting on the 20 kg mass is 40 N.

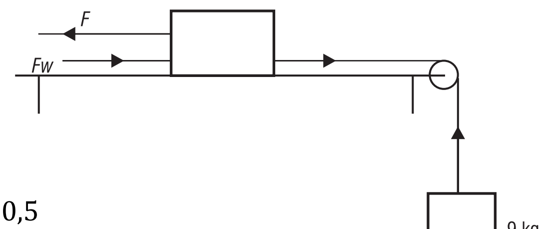


Figure 3.2

Solution:

$$F = ma$$
$$F - 40 - 90 = (20+9)0{,}5$$
$$F = 130 + 14{,}5$$
$$= 144{,}5\text{N}$$

Worked Example 3.4

Calculate the tension in the cable supporting a lift with a mass of 500kg, when the lift is accelerated:

(a) upwards at 0,3 m.s^{-2}

(b) downwards at 0,2 m.s^{-2}

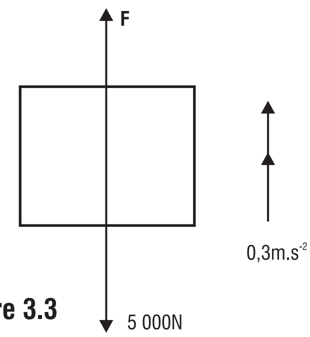


Figure 3.3

Solution:

(a) Because the acceleration is upwards, $F > 5\times10^3 N$

∴ The accelerating force is $F - 5\ \times 10^3 N$

From $F = ma$:

$$F - 5\ \times 10^3 = ma$$
$$F - 5\ \times 10^3 = 500(0{,}3)$$
$$F = 5\times10^3 \mid 150$$
$$= 5\ 150\ \text{N}$$

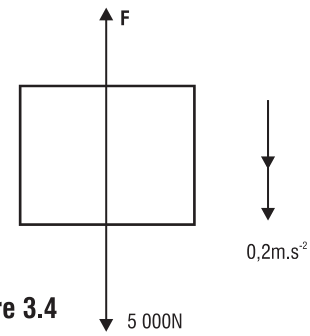


Figure 3.4

(b) The acceleration is downwards, therefore

$5\times10^3 N > F$

∴ The accelerating force is $5\ \times 10^3 - F$

∴ From $F = ma$

$$5\ \times 10^3 - F = ma$$
$$5\ \times 10^3 - F = 500(0{,}2)$$
$$F = 5000 - 100$$
$$= 4\ 900\ \text{N}$$

3.3 Work done

If work is to be done, energy must be available. Energy is described as the ability to do work. If a force is displaced, this is an indication that energy has been transferred and work done.

Work done is the product of the force and the displacement of the point of application of the force in the direction of the force.

Work done = force (F)x distance (s).

Work done and energy consist only of magnitude and are scalar quantities. The unit is N.m or J. The rate at which work is done is very important. It is called power, and is the work done per time unit or the energy transferred per time unit.

$$Power = \frac{force\ (F) \times distance(s)}{time(t)}$$

And is measured in J/s, or watt(W). The watt is the power that causes the production of energy at a rate of 1 J/s.

Worked Example 3.5

A body with a. mass of 10 kg is at rest on a. horizontal plane. A horizontal force of 12 N is applied to it and the resistance against motion is 6 N. The body is displaced through 40 m during 20 s.

Calculate:

(a) the work done by the resistance

(b) the work transformed into kinetic energy

(c) the total work done

(d) the total power

(Kinetic energy is the energy that a body contains because of its motion. See **Figure 3.3**)

Solution: $m = 10\ kg;\ u = 0\ m.s^{-1} - 1;\ F = 12\ N;$

$F_\mu = 6\ N;$

$a = 40\ m;\ t = 20\ s.$

(a) Work done $= F_\mu \times s$

$= 6(40)$

$= 240$ J

(b) Work done $= F.s$

$= (12 - 6)40$

$= 240$ J

(c) Total work done $= 240 + 240$

$= 480$ J

or work done $= 12(40)$

$= 480$ J

(d) Power $= \frac{work\ done}{time}$

$= \frac{480}{20}$

$= 24$ W

Worked Example 3.6

A spiral spring is 100 mm long when it is not elongated. A force of 5 N elongates the spring by 10 mm. Calculate the work done when the spring is elongated from 120mm to 150mm, if the limit of elasticity is not exceeded.

Solution:

To elongate the spring to a. length of 120mm a force of 2 x 5 = 10 N is required. When a spring has a length of 150mm, a force of 5 X 5 = 25 N is required.

Average force $= \frac{10+25}{2}$

$= 17{,}5$ N

Work done $= F\ x\ s$

$= 17{,}5 \text{ x } 0{,}03$

$= 0{,}525$ J

Worked Example 3.7

Calculate the average power exerted by an engine if it lifts a mass of 200 kg through 5m during 50 s.

Solution:

$m = 200\ kg;\ s = 5\ m;\ t = 50\ s.$

Work done $= F\ x\ s$

$= 2000 \text{ x } 5$

$= 10\ 000$ J

Power $= \frac{10\ 000}{50}$

$= 200$ W

Worked Example 3.8

A vehicle with a mass of 800kg, accelerates uniformly from rest to 60km/h during 5s up an incline of 1:20. The frictional resistance 250N.

Calculate:

(a) the work done against the frictional force when the velocity has reached 60km/h

(b) the work done against the gravitational component parallel to the plane

(c) the work transformed into kinetic energy

(d) the total force exerted on the vehicle

(e) the total power

Solution:

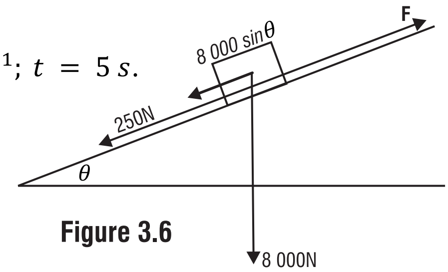


Figure 3.6

(a) $u = 0;\ v = 60\ km/h = 16{,}67\ m.s^{-1};\ t = 5\ s.$

$\text{Tan}\theta = \frac{1}{20}$

$\theta = \tan\frac{-1}{20}$

$= 2{,}86°$

and $v = u + at$

$16{,}67 = 0 + a(5)$

$a = 3{,}33\ \text{m.s}^{-2}$

Further: $s = ut + \frac{1}{2}at^2$

$= 0(5) + \frac{1}{2}(3{,}33)5^2$

$= 41{,}66\text{m}$

Work done $= F\ x\ s$

$= 250 \text{ x } 41{,}66$

$= 10{,}416 \text{ kJ}$

(b) Gravitational component parallel to the plane

$= 800 \sin 2{,}86°$

$= 39{,}92 \text{ N}$

Work done $= 39{,}92 \text{ x } 41{,}66$

$= 16{,}6 \text{ kJ}$

(c) Accelerating force $= ma$

$= 800\ (3{,}33)$

$= 2664 \text{ N}$

Work done $= F\ x\ s$

$= 2\ 664 \text{ x } 41{,}66$

$= 110{,}98 \text{ kJ}$

(d) Because the object is moving upwards, the resulting force is $F - 399{,}2 - 250$

$F - 399{,}2 - 250 = ma$

$F = 649{,}166 + 2\ 664$

$= 3\ 313{,}17 \text{ N}$

(e) Power $= \frac{work\ done}{time}$

$= \frac{3\ 313{,}17 \times 41{,}66}{5}$

$= 27{,}6 \text{ kW}$

3.4 Energy

Energy is the ability to do work.

3.4.1 Kinetic energy

The energy a body possesses due to its motion is called kinetic energy. A car moving at $20 m.s^{-1}$ possesses more kinetic energy than at any lower velocity. The unit in which energy is measured is the joule (J) . When a body is at rest, it possesses no kinetic energy.

A force (F) acts on an object of mass (m) and causes a displacement (s) during a time (t) as well as an acceleration (a). The force is doing work and energy is transferred to the body. The body possesses kinetic energy.

Work done = Force x distance

= $F \, x \, s$ ....(1)

= $m \, x \, a \, x \, s$ (because $F = ma$)... (2)

Assume that the object is initially at rest

$a = \frac{v-u}{t}$ ....(3)

$= \frac{v-0}{t}$

$a = \frac{u+v}{2} . t$

$= \frac{0+v}{2} . t$

$= \frac{v}{2} . t$ ....(4)

Substitute (3) and (4) into (2)

Work done = $m \times a \times s$

$= m \times \frac{v}{t} \times \frac{v}{2} \times t$

$= \frac{1}{2} mv^2$

The quantity $\frac{1}{2} mv^2$ is called the kinetic energy, E_k of the object. Work done is a measure of energy transferred, therefore energy is also measured in joules.

3.4.2 Potential energy

Potential energy is the energy a body possesses because of gravity and its relative position with respect to a reference plane.

The higher a body is above a reference plane the more potential energy it possesses with respect to the reference plane.

When a body mass (m) is raised vertically through a distance (h), work is done and energy is transferred to the object. If the object is to be raised at a constant velocity, a force (F) equal and opposite to the weight of the object is required (mg).

The energy transferred is:
Energy = work done
= $F . h$
= mgh

The energy transferred to the body is called potential energy (E_p). Potential energy (because of gravity) depends on the relative position of the system, such as that of an object with respect to the earth. In calculating change in potential energy, the vertical height between two positions is of importance, for example the distance between the floor and the table or the ground floor and the third floor of a building.

3.4.3 Conservation of mechanical energy

Energy cannot be destroyed or created, but can be changed from one form to another. Potential energy can be changed to kinetic energy.

Consider a stationary object that possesses 500 J of energy because of its height, 3m above a reference plane. The 500 J is potential energy. If the object is then allowed to fall free (and losses, such as the conversion of kinetic energy into heat are ignored,) this potential energy will be transformed entirely into kinetic energy.

Just as the body reaches the reference plane the potential energy will be nil and the kinetic energy 500 J.

3.3.4 Summary of formulae

- $E_k = \frac{1}{2} mv^2$
- $E_p = mgh$

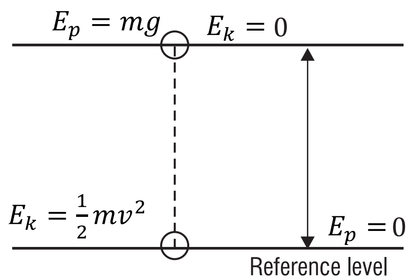


Figure 3.7

Worked Example 3.9

A body with a mass of 80 kg is lifted vertically through I2 m from A to B, and is then allowed to fall from B to A.

Calculate:

(a) the gain in potential energy when the body is lifted from A to B

(b) the kinetic energy just as the body reaches the point A when it is falling free from B to A

(c) the velocity when it reaches A.

Solution: $m = 80\ kg;\ h = 12\ m$

(a) $E_p = mgh$
$= 80(10)12$
$= 9{,}6$ kJ

(b) Gain in E_k = loss in E_p
$= 9{,}6$ kJ

(c) $\frac{1}{2}mv^2 = 9\ 600$
$\frac{1}{2}(80)v^2 = 9\ 600$
$v^2 = 240$
$v^2 = 15{,}49$ m.s^{-1}

Worked Example 3.10

A ball of mass 5 kg rolls from rest down a frictionless plane of 1:20 for 15 m.

Calculate:

(a) the loss in potential energy after 4 m

(b) the velocity after 15 m

(c) the gain in kinetic energy after 15 m.

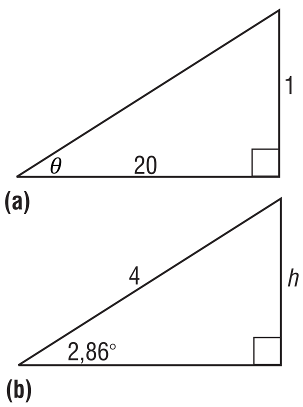


Figure 3.8

Solution: $m = 5\ kg;\ u = 0;\ s = 15m$

(a) From **Figure 3.8a**:

$\tan\theta = \frac{1}{20}$

$\theta = \tan^{-1}\frac{1}{20}$

$= 2{,}862°$

(b) From **Figure 3.8b**:

$\frac{h}{4} = \sin 2{,}862°$

$h = 4(0{,}0499)$

$= 0{,}2$ [for small angles, $\sin\theta = \tan\theta$ $\quad 0{,}049\ 9 = \frac{1}{20}$]

$E_p = mgh$

$= 5(10)0{,}2$

$= 10$ kJ

(b) $\frac{1}{2}mv^2 = 37{,}5$

$v^2 = \frac{37{,}5 \times 2}{5}$

$v = 15$ m.s^{-1}

(c) Loss in $E_p = mgh$

$= 5(10)\left(15 \times \frac{1}{20}\right)$

$= 37{,}5$ J

= gain in E_k

Worked Example 3.11

A train with a mass of 300 Mg accelerates uniformly up an incline of 1:100 from 10 km/h to a velocity of 50 km/h in 320m of travel. Use the law of conservation of energy, and calculate the work done by the locomotive as well as the tractive force when the rolling resistance is 35 kN.

Solution: $m = 3 \times 10^5\ kg;\ u = 10\ km/h = 2{,}778\ m.s^{-1};$
$v = 50\ km/h = 13{,}889\ m.s^{-1};\ 8 = 320m;$
F_μ = 35 kN

The work done by the train = gain in E_p + gain in E_k + work done against resistance.

$= mgh + \frac{1}{2}m(v^2 - u^2) + F_\mu \times s$
$= 3 \times 10^5(10)\left(320 \times \frac{1}{100}\right) + \frac{1}{2}(3 \times 10^5)$
$(13{,}889^2 - 2{,}778^2) + 35\ 000 \times 320$
$= 96 \times 10^5 + 277{,}78 \times 10^5 + 112 \times 10^5$
= 48,578 X 10^6 J
= 48,578 MJ

Work done $= force\ x\ distance$
48,578 x 10^6 = F x 320
F = 151,8 kN

Worked Example 3.12

A ball with a mass of 3 kg swings at the end of a string and describes a vertical circle with a radius of 3,5 m. The speed of the ball when it passes the lowest point is 9 $m.s^{-1}$.
Calculate:
(a) the kinetic energy of the ball when it reaches the lowest point
(b) the maximum height that it will reach when the lowest point is taken as the reference plane.

Solution:
(a) $E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(3)9^2$
$= 121{,}5J$

(b) Gain in E_p = loss in E_k
$mgh = \frac{1}{2}mv^2$
$3(10)h = 121{,}5$
$h = 4{,}05m$

or $4{,}05m$ above the lowest point

Worked Example 3.13

A hammer with a mass of 1,5 kg falls free, from rest, through a height of 1,2m and drives a nail into a wooden block.
Calculate:
(a) the velocity of the hammer just before it hits the nail
(b) the resistance of the wood if the nail is driven 15 mm into the block.

Solution: $m = 1{,}5\ kg;\ u = 0;\ 8 = 1{,}2\ m;\ g = 10\ m.s^{-2}$

(a)

$$v^2 - u^2 = 2gs$$
$$v^2 = 2(10)\ 1{,}2$$
$$= 24$$
$$v = 4{,}9\ \text{m.s}^{-1}$$

Or:

$$\text{gain in } E_k = \text{loss in } E_p$$
$$\frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}(1{,}5)v^2 = 1{,}5(10)1{,}2$$
$$v^2 = \frac{2(1{,}5)(10)1{,}2}{1{,}5}$$
$$= 4{,}9\ \text{m.s}^{-1}$$

(b)

$$\text{Work done on nail} = \text{loss in } E_k$$
$$F\,x\,s = \frac{1}{2}mv^2$$
$$F \times (0{,}015) = \frac{1}{2}(1{,}5)4{,}9^2$$
$$F = \frac{1{,}5 \times 0{,}015(4{,}9)^2}{2}$$
$$= 1{,}2\text{kN}$$

Activity 3.1

1. A car is moving on a horizontal road at a velocity of 15 $m.s^{-1}$ The brakes are applied and it comes to rest after a distance of 60 m. The frictional resistance is 200N and the mass of the car is 850 kg.
   Calculate:
   (a) the deceleration [1,875$m.s^{-2}$]
   (b) the braking force [1,39 kN]
   (c) the time taken to bring the car to rest

2. A vehicle with a mass of 800 kg is braked and brought to rest on a horizontal road over a distance of 80 m during 5 s
   Calculate:
   (a) the deceleration (two equations with two unknowns) [6,4$m.s^{-2}$]
   (b) the Initial velocity of the vehicle [32$m.s^{-1}$]
   (c) the braking force. [5,12kN]

3. A truck is moving in a straight line on a horizontal road at 70 km/h and is brought to rest during 8 s by a braking force of 15kN. Calculate the mass of the truck if the frictional force is 5kN. [8 230kg]

4. Calculate the force exerted by a man with mass of 80kg on the floor of a lift when the lift is accelerated.
   (a) upwards at 0,4 $m.s^{-2}$ [832N]
   (b) downwards at 0,5 $m.s^{-2}$ [760N]

Continued overleaf ...

5. The mass of a lift is 500 kg. Calculate the tension in the cable supporting the lift when the lift is:
   (a) accelerated upwards at 1,2 $m.s^{-2}$ [5,6kN]
   (b) accelerated downwards at 1 2 [4,4kN]
   (c) moving at a constant velocity [5 000N]
6. A light rope passes over a frictionless pulley and each end is tied to a 6kg mass. A mass of 1kg Is now added to one of the 6g masses.
   Calculate:
   (a) the acceleration of the mass [0,769 $m.s^{-2}$)
   (b) the tension in the rope. [64,62 N]
7. Calculate the maximum acceleration by which a casting with a mass of 500k be lifted if a tension of 6 kN in the cable must not be exceeded. [2$m.s^{-2}$)

8. A light rope is fixed to a 50 k g mass and passes over a fixed, frictionless pulley A mass of 80 kg is fixed to a rope that is connected to a movable pulley. Calculate the tensions and accelerations in both ropes (Hint: The downward acceleration of the 50 kg mass equals twice the upward acceleration of the 80 kg mass.) [50kg: 1,43 $m.s^{-2}$; 428,5 N; 80 kg: 0,715 $m.s^{-2}$; 857N]

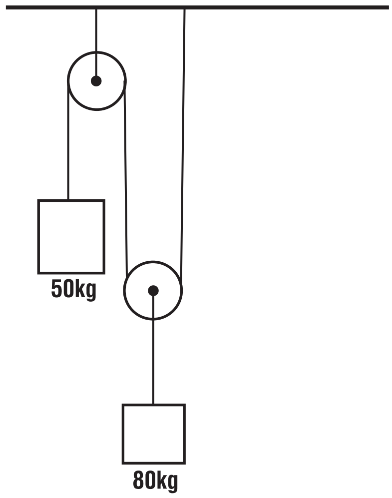


Figure 3.5

9. A train with a mass of 200 Mg is accelerated uniformly over a horizontal plane. The resistance against motion is 6N/Mg of the mass of the train and the tension in the tie bar is 35 kN.
   Calculate:
   (a) the force that will pull the train at a constant speed [1,2 kN]
   (b) the accelerating force [33,8 kN]
   (c) the acceleration. [0,169 $m.s^{-2}$]

10. A vehicle with a mass of 80 kg, accelerates in a straight line and uniformly up an incline of 15° from 2$m.s^{-1}$ to 12$m.s^{-1}$ during 20 a. The resistance against motion is 200 N.
   Calculate:
   (a) the acceleration [0,5$m.s^{-2}$]
   (b) the distance covered when 'the vehicle has reached a velocity of 12$m.s^{-1}$ [140m]
   (c) the resulting accelerating force [400 N]
   (d) the total force. [447 N]

11. A vehicle, mass 900 kg, accelerates uniformly and in a straight line up an incline of 8° from rest to 12 $m.s^{-1}$ over a distance of 300 m. Calculate the force required when the frictional force is 200 N. [1 668,6 N upwards]

Activity 3.2

1. A body with a mass of 8 kg is lifted upwards with a uniform velocity through 6m during 5s.
   Calculate:
   (a) the work done [480 J]
   (b) the power [96 W]
2. An object with a mass of 20 kg is at rest on a horizontal plane. A horizontal force of 50 N is applied to displace the object horizontally for 80 m. The resistance against motion is 20 N.
   Calculate:
   (a) the work done against the force [1,6 kJ]
   (b) the total work done frictional [4 kJ]
   (c) the work that is transformed into kinetic energy [2,4 kJ]
   (d) the total power when the force is applied for 10 s. (400 W]
3. A motor with an efficiency of 80 per cent operates a crane with an efficiency of 60 per cent. Calculate the velocity at which the crane lifts a mass of 600 kg when the power of the motor is 5 kW. (Hint: Power = force x velocity) [0,4 $m.s^{-1}$]
4. A mass of 8 kg moves (because of gravity) from rest down a rough inclined plane that makes an angle of 30° with the horizontal. The frictional force is 17,52 N and the mass reaches a velocity of 15 $m.s^{-1}$ in a distance of 40 m.
   Calculate:
   (a) the acceleration [2,81 $m.s^{-2}$]
   (b) the accelerating force [22,48 N]
   (c) the work done by the accelerating force [899,2 J]
   (d) the work done against the frictional force (700,8 J]
   (e) the total work done. (1,6 kJ]
5. Calculate the work done by gravity when a mass of 12 kg falls free for 30m. [3,6 kJ]
6. A spiral spring is 150 mm in length. A force of 8 N elongates it by 10mm. Calculate the work done when the spring is elongated from 160mm to 210mm. The limit of elasticity is not exceeded. (1,4 J]
7. A pushing force of 90 N displaces a mass of 30 kg at a constant velocity through 40m. The total frictional force is 60 N.
   Calculate:
   (a) the angle that the pushing force must make with the horizontal to ensure the movement (48,19°)
   (b) the work done by the horizontal component of the 90 N force [2,4 kJ]
   (c) the total work done. [3,6 kJ]
8. An object with a mass of 40 kg is resting on a horizontal plane. A force of 80 N at an angle of 20° to the horizontal pulls the object through 30 m at constant velocity.
   Calculate:
   (a) the frictional force (75,18 N]
   (b) the work done by the frictional force [2,255 kJ]
   (c) the total work done on the mass. [2,4 kJ]
9. Calculate the work done by gravity when a mass of 15 kg moves through a distance of 20 m on an inclined plane that makes an angle of 30° with the horizontal. The frictional force is 20 N. [1,1 kJ]
10. Calculate the magnitude of the frictional force if a 30 kW engine is needed to pull a mass at a constant velocity of 12 $m.s^{-1}$ over a horizontal plane. [2,5kN]
11. Calculate the power that will pull a mass of 90 kg at a constant velocity of 2 $m.s^{-1}$ down an inclined plane that makes an angle of 10° with the horizontal. The frictional force is 250 N. [187,44 W]
12. A car with a mass of 850 kg is standing on an incline of 1:15. The frictional force is 400 N. The brakes are released and the car moves down the incline.
    Calculate:
    (a) the work done by the gravitational component parallel to the plane when the car has travelled 40 m [22,62 kJ]
    (b) the work done against friction when 40 m has been covered [16 kJ]
    (c) the work transformed into kinetic energy.

Activity 3.3

1. A body with a mass of 40 kg, at rest 30 m above the ground, is allowed to fall free. Calculate;
   (a) the potential energy when it is 20 m above the round [8 kJ]
   (b) the kinetic energy when it is 20 m above the ground [4 kJ]
   (c) the kinetic energy just before it hits the ground [12 kJ]
   (d) the velocity just before it hits the ground. [24,49 $m.s^{-1}$]
2. A hammer is used to drive a nail into a wooden block. It strikes the nail at a velocity of 7 $m.s^{-1}$ and drives the nail 20 mm into the block. Calculate the resistance of the block if the mass of the hammer is 2 kg. [2,45 kN]
3. A mass of 5 kg falls free from rest and reaches a velocity of 12 $m.s^{-1}$ just before it hits the ground.
   Calculate:
   (a) the potential energy when the mass is still at rest [360 J]
   (b) the height through which it falls.[7,2m]
4. A car with a mass of 600 kg moves from rest down a smooth, inclined plane of 1:15 and covers a distance of 30m.
   Calculate:
   (a) the loss in potential energy after 30 m of travel [12 kJ]
   (b) the gain in kinetic energy after 30 m of travel [12 kJ]
   (c) the speed after 30 m of travel. [6,32 $m.s^{-1}$]
5. A ball with a mass of 2 kg is attached to a light rope that is 4 m long. The rope is anchored in such a way that the ball can describe a circle with a radius of 4 m in a vertical plane. The ball is held in a position 4 m due east of the anchoring point and then released.
   Calculate;
   (a) the kinetic energy as the ball passes the lowest point [80 J]
   (b) the velocity of the ball as it passes the lowest point. [8,94 $m.s^{-1}$]
6. A truck with a mass of 7 000 kg is at rest on an inclined plane of 1:30. The brakes are released and the truck moves down the incline. Calculate the velocity of the truck when 400 m has been covered, if the resistance against motion is 500 N. Also calculate the acceleration. [14,47 $m.s^{-1}$; 0,262$m.s^{-2}$]
7. A train with a mass of 350 Mg accelerates from 2 km/h up an incline that is at an angle of 4° to the horizontal, reaching 60 km/h in a distance of 400m. The resistance against motion is 50 kN.
   Calculate:
   (a) the work done by the locomotive [166,22 mJ]
   (b) the tractive force exerted by the locomotive. [415,6 kN]
8. A ball with a mass of 4 kg is projected up a smooth incline of 1:60 at 8 $m.s^{-1}$.
   Calculate:
   (a) the distance travelled when it comes to rest [192m]
   (b) the potential energy after the ball has covered 40 m. [26,67 J]
9. A mass of 6 kg is projected vertically upwards at a velocity of 8 $m.s^{-1}$.
   Calculate:
   (a) the maximum height it will reach [3,2 m]
   (b) the kinetic energy when it is 2 m above the ground [72 J]
   (c) the velocity when it is 2 m above the ground. [4,9 $m.s^{-1}$]
10. A car with a mass of 600 kg possesses 3,5 kJ kinetic energy and is accelerated horizontally over a smooth plane until it possesses 4,8 kJ kinetic energy, at which point it has travelled 120 m.
    Calculate:
    (a) the acceleration [0,018 $m.s^{-2}$]
    (b) the distance that it will travel up a smooth incline of 1:50 before it is brought to rest by the incline. (40 m)

Self Check

I am able to:

	YES	NO
• State Newton's three laws of motion and calculate the total force required for motion on horizontal and inclined planes with regards to practical problems, including gravitational force, frictional force and inertia.	○	○
• Calculate the work done over a given distance or during a given time and calculate the power at a given instant or at a given velocity.	○	○
• Define the concepts kinetic and potential energy and state the law of conservation of energy	○	○
• Apply the law of conservation of energy to a free-falling body and to motion on an inclined plane where no external force is applied.	○	○

If you have answered "no" to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Notes

Module 4 Statics

Learning Outcomes

On the completion of this module the student must be able to:

- Calculate the reactions at the supports of beams subjected to vertical point loads and evenly distributed loads. The mass of the beam is also considered as an evenly distributed load.
- Draw shear force and bending moment diagrams for simply supported beams subjected to point loads and evenly distributed loads.
- Determine the positions of the maximum bending moments and their magnitudes.
- Calculate the centroids of laminae and centres of gravity of solid objects by choosing the moment axis.

4.1 Introduction

Statics is the section of mechanics which studies forces that are not stationary or in other words not in motion.

4.2 Beams

For the purpose of this syllabus we will only consider beams with a constant cross-sectional area. Such a beam is called a uniform beam.

The centre of gravity of a uniform beam is at its midpoint and is the point on which the gravitational force is acting. When we talk of a light beam it means that the mass is negligible and can be ignored.

Beams can be subjected to concentrated loads (point loads), or it can be subjected to uniformly distributed loads.

4.2.1 Concentrated loads

The application of a single force may be considered to occur at a point which theoretically has no area. This, of course, is physically impossible.

There must be a finite area of contact - even the point of a needle has a finite (and therefore measurable) area, albeit very small.

From a mathematical point such a rationalization is not only possible and justifiable, but it can also be extremely convenient.

Any load that is applied to a relatively small area may be considered as a concentrated load.

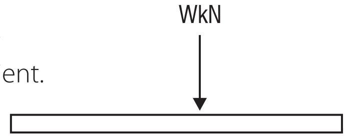


Figure 4.1

A concentrated or point load is therefore one which is applied to the beam by a knife-edge, that is, the load is not spread over any measurable part of the span.

4.2.2 Uniformly distributed loads

Loads which are continuous and spread over the longitudinal axis of a beam are treated as distributed loads, that is, the load is distributed over the length of the beam and measured in terms of mass per unit length.

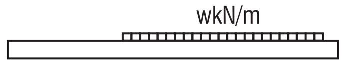


Figure 4.2

A uniformly distributed load is therefore one which is applied evenly over the whole or part of the beam so that the load carried by any portion of the beam under that load is proportional to the length of that portion.

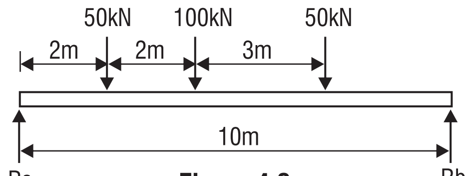


Figure 4.3

4.2.3 Simple supported beams

Beams usually rest on two supports, also known as reactions. The tendency of the applied forces (loads) is to cause the beam to move. The fixed points or supports react against this tendency and so the forces generated in the supports are called reactions.

If the forces are applied to a horizontal beam as shown in **Figure 4.3** the beam could be held in equilibrium by the application of the reactions which are equal and opposite to the resultant of the downward forces. The reactions Ra and Rb must both be vertical, since there is no horizontal force component.

Furthermore, in order to satisfy the condition of static equilibrium (that the sum of the vertical forces must be zero) the sum Ra + Rb must be equal to the sum of the downward-acting forces.

Sum of vertical forces = 0

$$\Sigma(V) = 0$$

$$\therefore R_a + R_b = 50 + 100 + 50 = 200\text{kN}$$

The magnitude of the reactions may now be found by the application of the third law of static equilibrium, namely that the algebraic sum of the moments of the forces about any point must be zero.

By taking moments about one of the unknown reactions the other reaction may be calculated as demonstrated by **Example 4.1.**

Worked Example 4.1

A uniform beam with a mass of 120kg is 8m long. It rests horizontally on two supports, one being 1m from the left-hand end and the other at the righthand end. The beam carries concentrated loads of 500N and 1200 N, 3m and 5m from the left end. It also carries an uniformly distributed load of 80N/m over the right half of the beam.

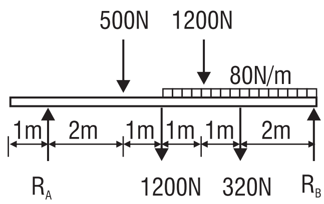


Figure 4.4

Calculate the reactions at the supports and test your answer.

Solution

Since the sum of the moments about A equals zero, the sum of the moments acting about A in a clockwise direction will be equal to the sum of the moments acting about A in an anticlockwise direction:

Take moments about A $\Sigma \circlearrowright M_A = \Sigma \circlearrowleft M_A$

$$500(2) + 1\,200(3) + 1\,200(4) + 320(5) = R_B 7$$

$$1\,000 + 3\,600 + 4\,800 + 1\,600 = 7\,R_B$$

$$R_B = 11\,000$$

$$\therefore R_B = 1\,571{,}43 \text{ N}$$

Take moments about B

$$\Sigma \circlearrowright M_B = \Sigma \circlearrowleft M_B$$

$$7\,R_A = 500(5) + 1\,200(4) + 1\,200(3) + 320(2)$$

$$= 2\,500 + 4\,800 + 3\,600 + 640$$

$$7\,R_A = 11\,540$$

$$\therefore R_A = 1\,648{,}53 \text{ N}$$

Continued overleaf...

Test

Since the sum of the vertical forces equals zero, the sum of the upward forces will be equal to the sum of the downward forces.

$$\Sigma(V)up = \Sigma(V)down$$
$$R_A + R_B = 1\ 000 + 3\ 600 + 4\ 800 + 1\ 600$$
$$\therefore 1\ 648{,}57 + 1\ 571{,}43 = 3\ 220$$
$$3\ 220 = 3\ 220$$

4.2.4 Cantilevers

A cantilever is a beam which is supported only at one end. Consider the cantilever A B shown in **Figure 4.5**. For equilibrium, the reaction force at A must be vertical and equal to the load W.

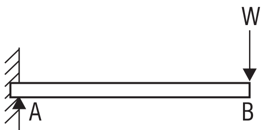


Figure 4.5

4.3 Shearing force and bending moment

Bending moment and shearing force are closely related to each other, as well as to the applied loading.

When a beam is subjected to lateral loading there is a bending moment and shearing force at each section of the beam which vary in intensity from section to section along the beam. The beam must transmit some proportion of the applied load to each of its supports.

This is achieved by the development of resistance to shearing forces within the fabric of the beam. The beam must also remain nearly straight and therefore has to develop resistance to bending.

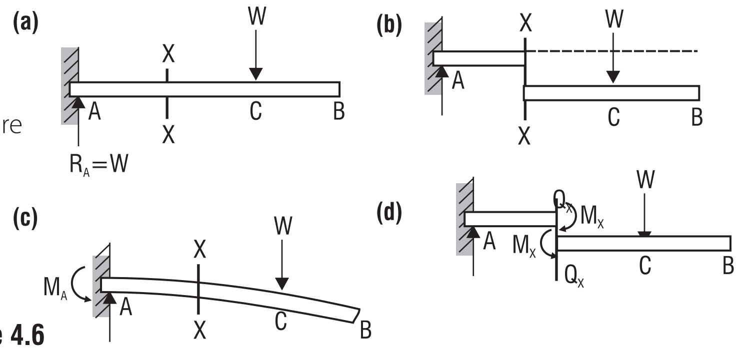


Figure 4.6

Consider the cantilever AB shown in **Figure 4.6a**. For equilibrium, the reaction force at A must be vertical and equal to the load W. The cantilever must therefore transmit the effect of load W to the support at A by developing resistance to the load effect called shearing force.

Failure to transmit the shearing force at any given section $x - x$ will cause the beam to fracture as in **Figure 4.6b**. Failure to resist the bending effect of the load sufficiently will cause the beam to deform as in **Figure 4.6c**.

The shearing force and the bending moment transmitted across the section $x - x$ (**Figure 4.6d**) may be considered as the force and moment respectively that are necessary to maintain equilibrium if a cut is made severing the beam at $x - x$.

The shearing force (Q_x) between A and C = W.
The bending moment (M_x) between A and C = W.

Note:
Both the shearing force and the bending moment will be zero between C and B.

4.4 Sign convention

Both shearing force and bending moment are vector quantities requiring a convention of signs for values of opposite meaning to be separated. When drawing shearing force and bending moment diagrams, always use the sign convention given in **Table 4.1**.

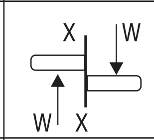
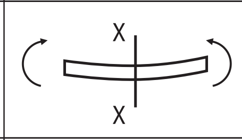
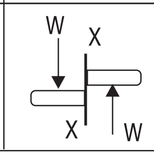
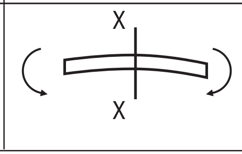
Sign	Shear force	Bending moment
+		
-		

Table 4.1

4.5 Definitions

Definition: Shearing force
The shearing force at any section of a beam is the algebraic sum of all the forces to the left or to the right of the section.

Definition: Bending moment
The bending moment at any section of a beam is the algebraic sum of the moments about the section of all the forces to the left or to the right of the section.

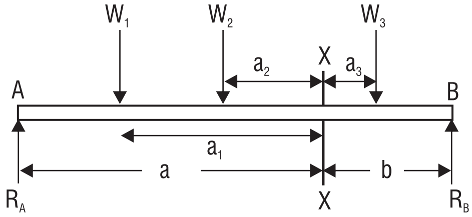


Figure 4.7

In the beam shown in **Figure 4.7**, the shearing force and bending moment at section $x - x$ can be calculated as follows:

Working to the left of $x - x$

Shearing force (Q_x) = $R_A - W_1 - W_2$

Bending moment (M_x) = $R_A a - W_1 a_1 - W_2 a_2$

Working to the right of $x - x$

$Q_x = -R_B + W_3$

$M_x = R_B b = W_3 a_3$

Note:
This rule is easily applied by starting at the left-hand end of the beam and plotting from there in the direction of the forces

4.6 Shearing force and bending moment diagrams

The following rules should be followed when drawing shearing force and bending moment diagrams for a conventional system of graphical representation to be adopted.

1. Draw the loading diagram, representing the distribution of loads.
2. Draw the shearing force diagram (SFD) directly below the loading diagram using the sign convention given in **Table 4.1**, plotting positive values above the line and negative values below the line.
3. For the bending moment diagram (BMD) calculate the bending moment at each important point on the loading diagram, such as a sudden change in the loading points where there is a sudden increase in loading and at the point where there is a maximum bending moment.
4. Mark the loading diagram, whether it is drawn to scale or not, with all the important values. Mark all positive and negative zones in the SFD.
5. Note that the diagrams are graphs and that the coordinates perpendicular to and measured from the base line give the value of the particular load effect at that section of the beam.

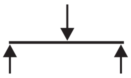
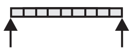
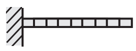
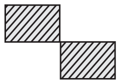
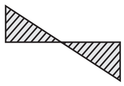
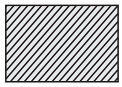
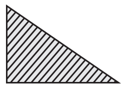
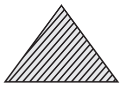
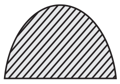
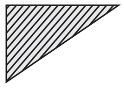
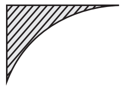
	Simple supported beam		Cantilever	
	Concentrated load	**Uniformly distributed load**	**End-point load**	**Uniformly distributed load**
Loading diagram				
Shearing force diagram				
Bending moment diagram				

Table 4.2

4.7 Standard beams and cantilevers

A number of particular loading cases are shown in **Table 4.2.**

4.8 Properties of curves and diagrams

Referring to **Table 4.2** the following properties can be observed.

1. The magnitude of a point load is represented by a vertical line in the shearing force diagram. The shear force diagram is a series of horizontal steps for any part of the span carrying point loads only.
2. For any part of the span carrying point loads only, the bending moment diagram is a series of sloping straight lines.
3. For any part of the span carrying a uniformly distributed load only, the shearing force diagram is a sloping straight line and the bending moment diagram is a parabola.
4. At the point where the shearing force diagram passes through zero the bending moment is either a maximum or a minimum (at this point the shear force sign changes).
5. The bending moment has a constant value, where the shearing force over any part of the span is zero.
6. The point of contraflexion or inflexion is where the bending moment diagram passes through zero (where the bending moment sign changes).

Worked Example 4.2

Draw the shearing force and bending moment diagrams for the following simple beam (**Figure 4.8**).

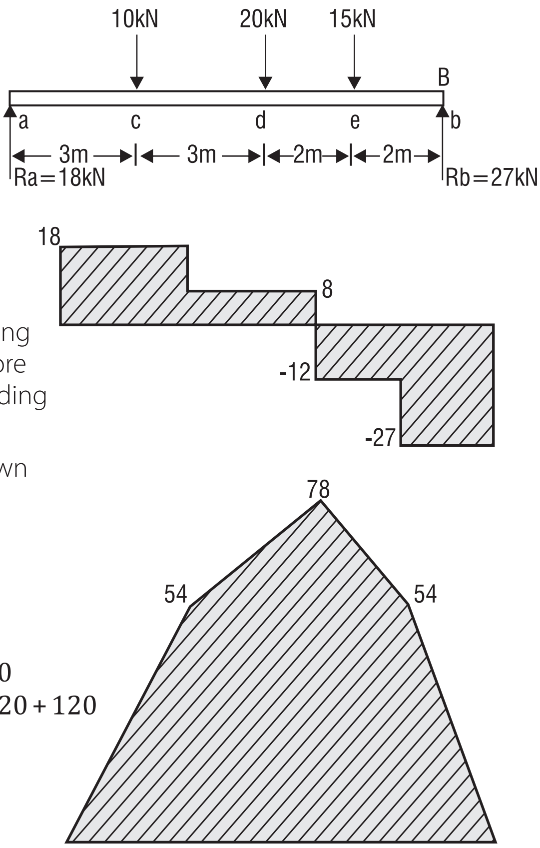


Figure 4.8

Solution:

It is convenient to draw the loading diagram to scale, leaving enough space to draw the shearing force and bending moment diagrams directly below it.

Then calculate the reactions and bending moments at each important point before plotting them in order to draw the bending moment diagram.

The shearing force diagram can be drawn directly from the values in the loading diagram.

Reactions

Taking moments about

$$R_a : \Sigma (M)_a = 0$$
$$(10 \times 3) + (20 \times 6) + (15 \times 8) = Rb \times 10$$
$$10\,R_b = 30 + 120 + 120$$
$$= \frac{270}{10}$$
$$\therefore R_b = 27\text{ kN}$$

Taking moments about

$$Rb : \Sigma (M)_b = 0$$
$$R_a \times 10 = (15 \times 2) + (20 \times 4) + (10 \times 7)$$
$$10\,R_a = 30 + 80 + 70$$
$$R_a = \frac{180}{10}$$
$$\therefore R_a = 18\text{ kN}$$

Bending moments

$$M_a = 0$$
$$M_c = 18 \times 3 = 54\text{ kN}$$
$$M_d = (18 \times 6) - (10 \times 3)$$

(sum of moments to the left of **d**)

$$= 108 - 30$$
$$= 78\text{ kN}$$

or $M_d = (27 \times 4) - (2 \times 15)$

(sum of moments to the right of **d**)

$$= 108 - 30$$
$$= 78\text{ kNm}$$
$$M_c = (18 \times 8) - (10 \times 5) - (20 \times 2)$$

(sum of moments to the left of **e**)

$$= 144 – 50 - 40$$
$$= 54\text{ kNm}$$

or $M_c = (27 \times 2)$

(sum of moments to the right of **e**)

$$= 54\text{ kNm}$$
$$M_b = 0$$

Worked Example 4.3

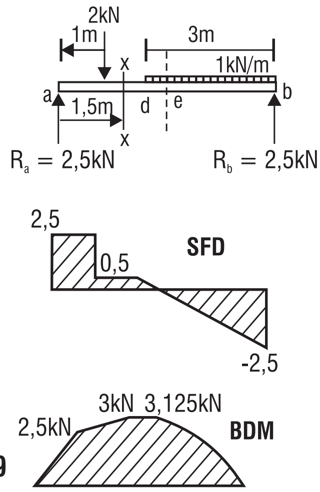


Figure 4.9

A simple beam 5 m long has a uniformly distributed load of 1 kN per metre extending over a length of 3 m from the right-hand reaction.

The beam also carries a concentrated load of 2 kN one metre from the lefthand reaction (**Figure 4.9**).

(a) Draw the shearing force and bending moment diagrams

(b) compute the maximum bending moment

(c) find the shearing force Qs and bending moment M,., 1,5 m from the lefthand reaction.

Solution:

(a) **Reactions:**

Taking moments about

$R_a : \Sigma (M)_a = 0$

$R_b \times 5 = (2 \times 1) + (3 \times 3{,}5)$

$= 2 + 10{,}5$

$5\ R_b = 12{,}5$ kN

$R_b = \frac{12{,}5}{5}$

$\therefore R_b = 2{,}5$ kN

Taking moments about

$R_b : \Sigma (M)_b = 0$

$R_a \times 5 = (1{,}5 \times 3) + (2 \times 4)$

$= 4{,}5 + 8$

$5\ R_b = 12{,}5$ kN

$R_b = \frac{12{,}5}{5}$

$\therefore R_b = 2{,}5$ kNm

Bending moments

$M_a = 0$

$M_c = (2{,}5 \times 1)$

$= 2{,}5$ kNm

$M_d = (2{,}5 \times 2) - (2 \times 1)$

$= 5 - 2$

$= 3$ kNm

$M_o = 0$

(b) The maximum bending moment occurs at e where the shear force is zero. Let this point be y metres from a. From the shear force diagram:

$de = (y - 2)\ m$

The force due to the distributed load of 1 kN over a length of $(y - 2)$ m is:

load $= 1\ x\ (y - 2)$

$= (y - 2)$ kN

For maximum bending moment the sum of the shearing force to the left of **e** is zero:

$\Sigma(Q)_c = 0$

$R_a - 2 - (y - 2) = 0$

$2{,}5 - 2 - (y - 2) = 0$

$0{,}5 - y + 2 = 0$

$y = 2{,}5$ m

$(M)_c = (R_a \times y) - 2(y - 1)$

$- (y - 2)\ \frac{(y - 2)}{2}$

$= (2{,}5 \times 2{,}5) - 2(2{,}5 - 1)$

$= -1(2{,}5 - 2)\ \frac{(2{,}5 - 2)}{2}$

$= 3{,}125$ kNm

Continued overleaf ...

(c) Shearing force at section ***x***:

$Q_x = R_a - 2$ (sum of shearing forces to the left of ***x***)

$= 2{,}5 - 2$

$= 0{,}5 \text{ kN}$

or $Q_x = - R_b + 3$ (sum of shearing forces to the right of ***x***)

$= 0{,}5 \text{ kN}$

Bending moment at section ***x***

Taking moments about ***x***; $\Sigma(M)_x = 0$

$M_x = Ra \times 1{,}5) - (2 \times 0{,}5)$

$= (2{,}5 \times 1{,}5) - (2 \times 0{,}5)$

$= 3{,}75 - 1$

$= 2{,}75 \text{ kNm}$

Worked Example 4.4

Calculate the reaction and draw the shearing force and bending moment diagrams for the beam shown in **Figure 4.10**.

Calculate the magnitude of the shearing force and bending moment at point F.

Solution:

Reaction at a:

$$\Sigma(Q)_a = 0$$
$$Ra = 10 + (20 \times 2{,}5)$$
$$= 10 + 50$$
$$Ra = 60 \text{ kN}$$

Bending moments:

$$\Sigma(M) = 0$$
$$M_a = -(20 \times 2,5) \times 2,75 - 10 \times 3,5$$
$$= -137{,}5 - 35$$
$$= -172{,}5 \text{ kNm}$$
$$M_b = -(20 \times 2{,}5) \times 1{,}25 - 10 \times 2$$
$$= -62{,}5 - 20$$
$$= -82{,}5 \text{ kNm}$$
$$M_c = -(20 \times 0{,}5) \times 0{,}25$$
$$= -2{,}5 \text{ kNm}$$
$$M_d = 0$$

Shearing force at F

$$Q = (20 \times 1{,}5) + 10$$
$$= 30 + 10$$
$$= 40 \text{kN}$$

Bending moment at F

$$M_F = -(10 \times 1,5) - (20 \times 2) \times 1$$
$$= -15 - 40$$
$$= -55 \text{ kNm}$$

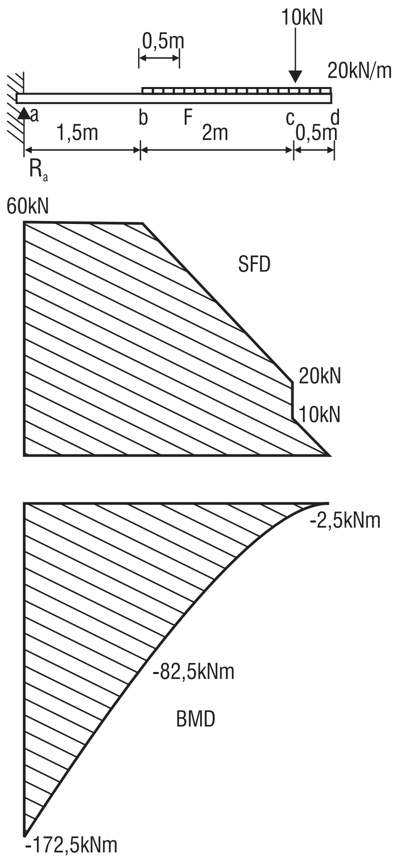


Figure 4.10

Note:
$M_{max} = M_a$

4.9 Centre of gravity and centroid

It can be assumed that any solid consists of a very great number of particles of mass (m).

Each of these particles will be subjected to gravity (mg). The weight (Mg) of a body is then built up by a great number of parallel forces {mg). The working line of this resultant, regardless of how the body is placed, always passes through one specific point, which is the position of the centre of gravity.

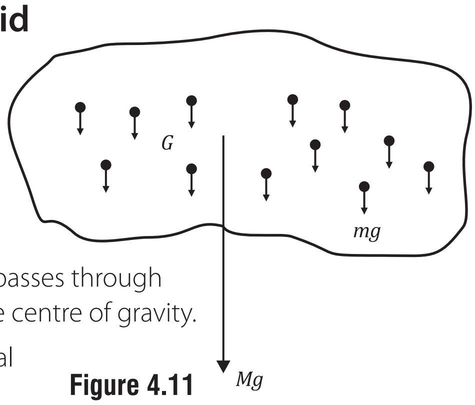


Figure 4.11

The point of application (G) of the gravitational force is called the centre of gravity.

The centre of gravity of a body is a point through which the working line of the weight of the body passes, irrespective of how the body is placed.

An area possesses no weight and has no centre of gravity. In this case the corresponding point is called the centroid.

The centre of gravity of a body is also called the centre of mass, because the centre of gravity is independent of the acceleration due to gravity and depends only on the mass.

4.9.1 Determining the centre of gravity by means of a plumbline

1. Make three small holes in a piece of cardboard, well-spaced and near the edge of the cardboard.
2. Pass a stout pin through one of the holes and hold it firmly by a clamp stand so that the card can swing freely on it. The card will come to rest with its centre of gravity vertically below the point of the support.
3. Fix a plumbline to the pin and mark the vertical line on the cardboard.
4. Repeat the process using the other two holes.
5. The three lines will meet at one point, which is the centre of gravity.

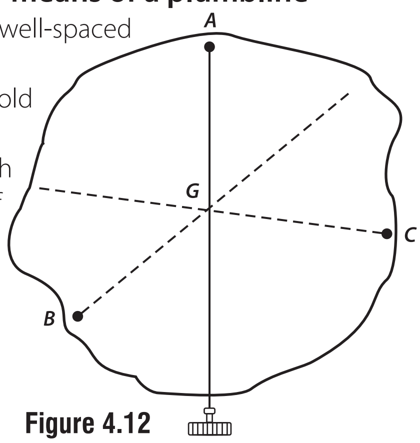


Figure 4.12

4.9.2 Centroids of laminae and centres of gravity of solids

A lamina is a thin plate that does not possess any thickness. When calculating the position of the centroid, only the area is taken into account.

The centroid of laminae and centres of gravity of solids in general use, are determined as follows.

- **A square**

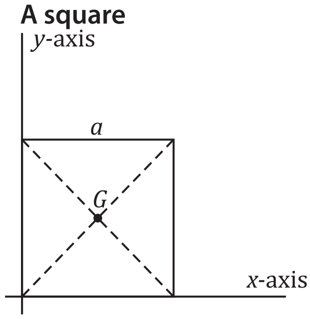


Figure 4.13

G is at $(\bar{x};\bar{y}) = (\frac{a}{2};\frac{a}{2})$

- **A rectangle**

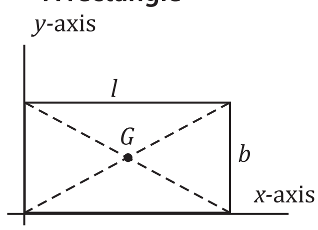


Figure 4.14

G is at $(\bar{x};\bar{y}) = (\frac{l}{2};\frac{b}{2})$

- **A disc**

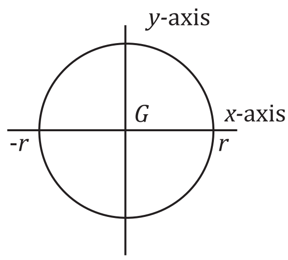


Figure 4.15

G is at $(\bar{x};\bar{y}) = (0;\ 0)$
or G is at $\bar{x} = \frac{D}{2} = r$
Where $D =$ diameter

- **A triangle**

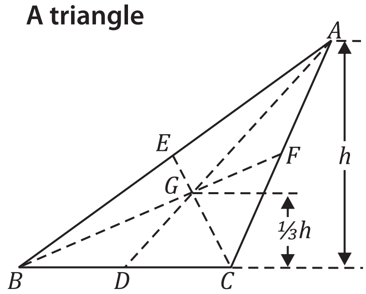


Figure 4.16

In a triangular plate or area, the centre of gravity is at the point of intersection of the lines that join the centre of the sides to the opposite angles.

$\bar{y} = \frac{1}{3}h$

- **A sphere**

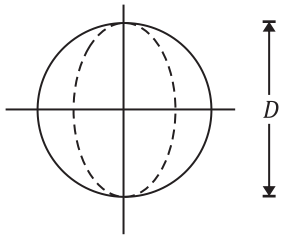
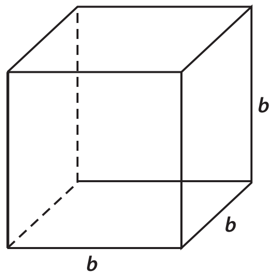


Figure 4.17

$(\bar{x}; \bar{y}) = (0; 0)$

or $\bar{x} = \frac{D}{2} = r$

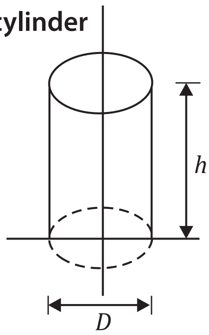
- **A cube**



G is at $(\bar{x}; \bar{y}) = (\frac{b}{2}; \frac{b}{2})$

Figure 4.18

- **A cylinder**



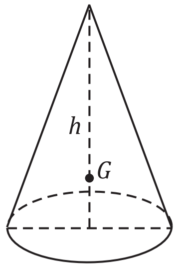
$(\bar{x}; \bar{y}) = (0; h/2)$

or $\bar{x} = \frac{D}{2} = r$

$y = \frac{1}{2}$

Figure 4.19

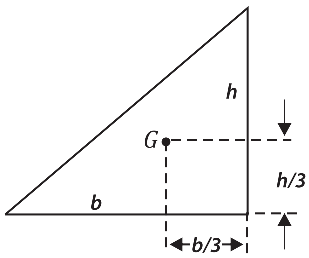
- **A cone**



$\bar{y} = \frac{1}{4}h; \ \bar{x} = 0$

Figure 4.20

- **A right-angled triangle**



$\bar{y} = \frac{1}{3}h$

Figure 4.21

- **A triangular prism (isosceles)**

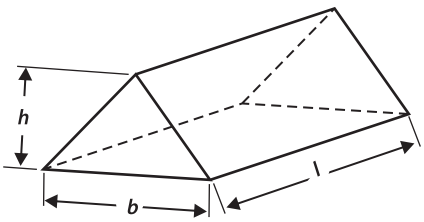


Figure 4.22

G is at the intersection of $\frac{h}{3}$; $\frac{b}{2}$ and $\frac{1}{2}$

4.10 Some formulae

- **Area**
  Rectangle: length x breadth
  Circle: πr^2 or $\frac{\pi D^2}{4}$
  Triangle: $\frac{1}{2}$ base x height
- **Volume**
  Rectangular prism: length x breadth x height
  Sphere: $\frac{4}{3}\pi r^3$
  Prism: area of base x perpendicular height
  Cone: $\frac{1}{3}\pi r^2 h$

Worked Example 4.5

Calculate the centroid of the lamina in **Figure 4.23**.

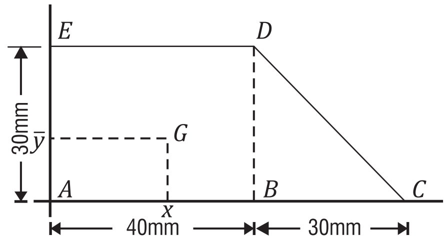


Figure 4.23

Rectangle ABDE:

Area = 40×30

= $1\,200 \text{ mm}^2$

Co-ordinates of the centroid = $(20;15)$

Triangle BCD:

Area = $½(30)(30)$

= 450 mm^2

Co-ordinates of centroid = $(50; \frac{1}{3}h)$

= $(50; 10)$

Total area = $1\,200 + 450$

= 1650 mm^2

Let the co-ordinates of the total lamina be $(\bar{x}; \bar{y})$

Take moments about the x-axis

Moment of the total area = the sum of the moments of the separate parts

$1\,650\bar{y} = 1\,200(15) + 450(10)$

$1\,650\bar{y} = 18\,000 + 4\,500$

$\bar{y} = 13{,}64 \text{ mm}$

Take moments about the y-axis

$1\,650\,\bar{x} = 1\,200(20) + 450(50)$

$1\,650\,\bar{x} = 24\,000 + 22\,500$

$\bar{x} = 28{,}18 \text{ mm}$

Worked Example 4.6

Calculate the centroid of the area shown in **Figure 4.24**.

Solution:

Part	Area in mm²	Co-ordinates of centroid
ABKL	20 x 60 = 1 200	(10;70)
CDIJ	60 x 20 = 1 200	(50;70)
DFGH	140 x 30 = 4 200	(95;70)
Total	**6 600**	$(\bar{x}; \bar{y})$

Table 4.3

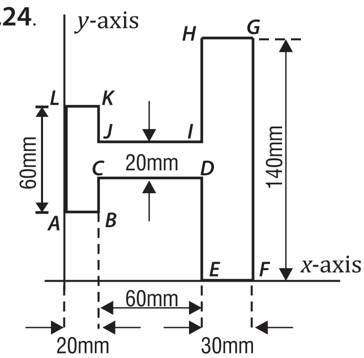


Figure 4.24

Take moments about the y-axis

Moment of the total area = E moments of the parts

$6\,600\,\bar{x} = 1\,200(10) + 1\,200(50) + 4\,200(95)$

$= 12\,000 + 60\,000 + 399\,000$

$\bar{x} = 71{,}36 \text{ mm}$

and $\bar{y} = 70 \text{ mm}$

Worked Example 4.7

Calculate the co-ordinates of the centroid of the plate shown in **Figure 4.25**.

Solution:

Take moments about the x-axis

Moment of total area = Σ moments of the parts

$$15{,}215\bar{y} = 12(2) + 4(\tfrac{4}{3}) - 0{,}785(2)$$
$$= 24 + 5{,}333 - 1{,}57$$
$$= 27{,}763$$
$$\bar{y} = 1{,}825\text{mm}$$

Take moments about the y-axis

Moment of total area = Σ moments of parts

$$15{,}215\bar{x} = 12(1,5) + 4(3,667) - 0{,}785(1{,}5)$$
$$= 18 + 14{,}688 - 1{,}178$$
$$= 31{,}49$$
$$\bar{x} = 2{,}07\text{ mm}$$

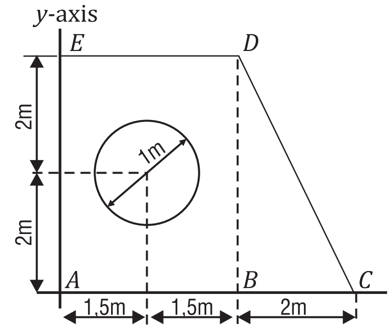


Figure 4.25

Part	Area in mm²	Co-ordinates of centroid
ABDE	3 x 4 = 12	(1;5;2)
ΔBCD	½(2)(4) = 4	$(3\frac{2}{3};\frac{4}{3})$
Circle	$\frac{-x\times1^2}{4}$ = -0,785	(1,5;2)
Total	**15 215**	$(\bar{x};\bar{y})$

Table 4.4

Worked Example 4.8

A triangular plate with the sides 60 mm long and the base 70 mm long has a hole, with an area 400 mm², with its centre on the gravitational line and 10 mm from the base. Calculate the distance of the centroid of the plate from the base.

Solution:

$$BD^2 = AD^2 - AB^2$$
$$= 602 - 352$$
$$= 3\,600 - 1\,225$$
$$= 2375$$
$$BD = 48{,}73\text{ mm}$$

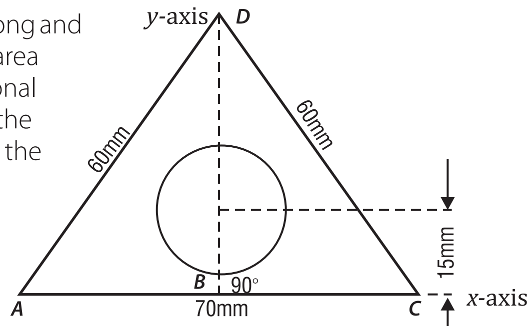


Figure 4.26

Part	Area in mm²	Co-ordinates of centroid
Triangle	½(70)(48,73) = 1 705,69	(0;16;24)
Circle	-400	(0;15)
Total	**1 305,69**	$(\bar{x};\bar{y})$

Table 4.5

Take moments about the x-axis

Moment of total area = Σ moments of the parts

$$1{,}305{,}69\,\bar{y} = 1705{,}69(16{,}24) - 400(15)$$
$$= 27\,700 - 6\,000$$
$$= 21\,700$$
$$\bar{y} = 16{,}62\text{ mm}$$

Worked Example 4.9

The pivot shown in **Figure 4.27** has a conical hole at one end. Calculate the x-ordinate of the centre of gravity of the pivot.

Solution:

With solids, volumes are calculated, which, if multiplied by the density, give the mass. Centres of gravity are independent of gravity but dependent on mass. Mass is dependent on volume.

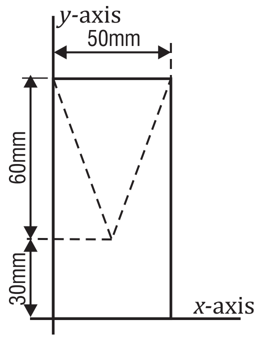


Figure 4.27

Part	Area in mm²	Co-ordinates of centroid
Cylinder Cone	$\pi(25)^2 90 = 1{,}77 \text{ x } 10^5$ $\frac{1}{2}\pi(25)^2 60 = -3{,}927 \text{ x } 10^4$	(25;245) (25;75)
Total	**$1\ 377 \text{ x } 10^5$**	$(\bar{x};\bar{y})$

Table 4.6

Take moments about the x-axis

Moment of total volume = Σ moments of the parts

$$1{,}377 \text{ x } 10^5 (\bar{y}) = 1{,}77 \text{ x } 10^5 \text{ x } (45) - 3{,}927 \text{ x } 10^4 (75)$$
$$= 7{,}97 \text{ x } 10^6 - 2{,}945 \text{ x } 10^6$$
$$= 5{,}025 \text{ x } 10^6$$
$$\bar{y} = 36{,}491 \text{ mm}$$

Activity 4.1

1. A beam with a mass of 80 kg carries a uniformly distributed load of 40 N/m. Calculate the magnitudes of the reactions P and Q when the system is in equilibrium. [P = 1105 N; Q = 355 N]

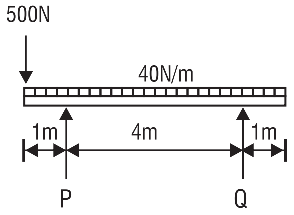


Figure 4.28

2. Calculate the magnitude and position of the concentrated load that will keep the beam in equilibrium. The mass of the beam is 80kg. [2 100 N 2,9m from right end]

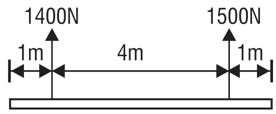


Figure 4.29

3. A uniform beam has a mass of 20 kg and is 5m long. It is supported at the left end and 1 m from the right end. It carries two concentrated loads, 20N at the right end and 100 N at a position that makes the two reaction forces equal. Calculate that position. [0,4 m from the left end]
4. A uniform beam has a mass of 50 kg and is 8 m long. It is supported at two points, 6m apart. The one support is at the left end.

   Calculate;

   (a) the position (between the supports) at which a concentrated load of 300 N must be applied if the reactions in the two supports are to be equal [At the centre of gravity of the beam]

   (b) the position (between the supports) at which a concentrated load of 2 000 N must be applied if the reactions in the two supports are to be equal [At the centre of gravity of the beam]

   (c) the position (between the supports) at which a concentrated load of 300 N must be applied if the reaction in the right-hand support is twice that in the left-hand support. [5,67 m from the support on the left]

5. (a) A simple beam 5 m long has a uniformly distributed load of 1 kN per metre extending over a length of 3 m from the right-hand reaction (**Figure 4.30**). Calculate the magnitude of the reactions. [0,9 kN; 2,1 kNJ

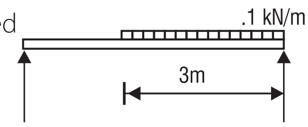


Figure 4.30

   (b) Explain the terms concentrated load uniformly distributed load and cantilever.

6. A cantilever 6 m long carries a load of 20 kN at the free end and 40 kN at 2 m from the fixed end. A uniformly distributed load of 10 kN/m covers the first 4 m from the fixed end (**Figure 4.31**). Calculate the reaction. [100 kN]

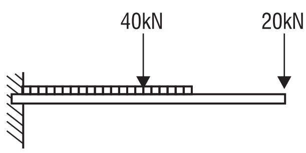


Figure 4.31

Activity 4.2

1. **Figure 4.32** shows the shear force diagram for a loaded beam. Deduce and sketch the loading diagram.

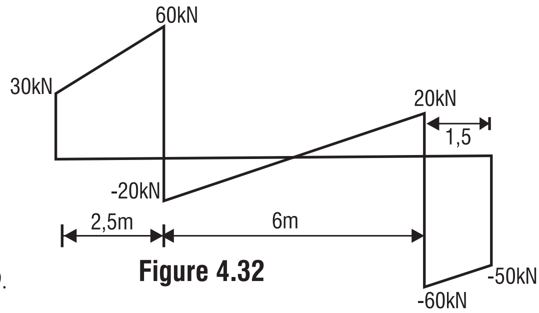


Figure 4.32

2. Calculate the reactions and draw the shearing force and bending moment diagrams for the beams shown in **Figure 4.33** – **Figure 4.39**. Calculate the magnitude of the shearing force and bending moment at point J in each case.

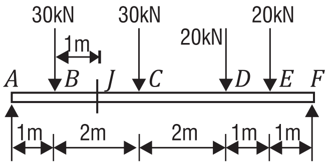


Figure 4.33

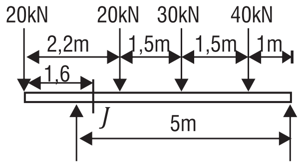


Figure 4.34

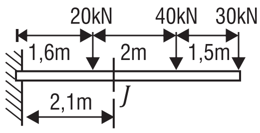


Figure 4.35

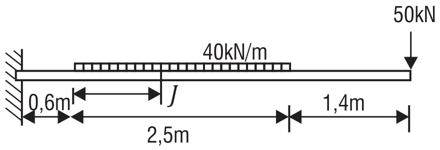


Figure 4.36

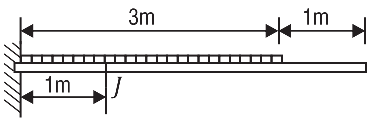


Figure 4.37

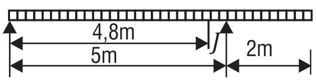


Figure 4.38

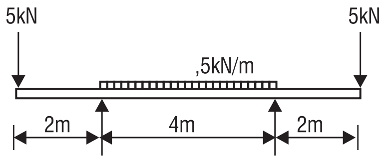


Figure 4.39

3. Draw the shearing force and bending moment diagrams for **Figure 4.40 – Figure 4.43**. Find the position and magnitude of the maximum bending moment in each case.

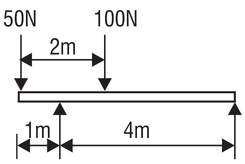


Figure 4.40

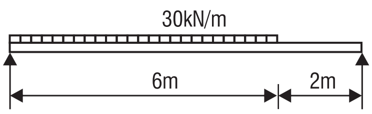


Figure 4.41

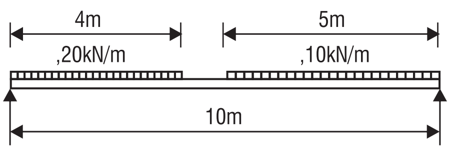


Figure 4.42

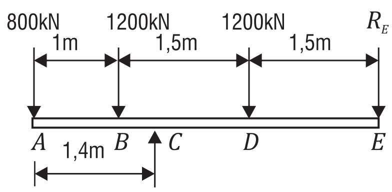


Figure 4.43

Continued overleaf ...

4. A beam with a uniformly cross sectional area is 8 m long and has a weight of 10 kN per linear metre. The beam is supported at points 2 m from each end. Draw the shear force and bending moment diagrams.
5. **Figure 4.44** shows a simple beam, supported at points 2 m from each end, carrying a point load of 10 kN at each end and a uniformly distributed load spread over the length between the supports. If the bending moment is zero at the mid-span, find the magnitude of the uniformly distributed load and sketch the shearing force and bending moment diagrams.

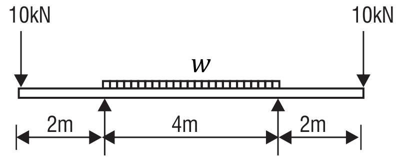


Figure 4.44

6. A simple supported beam 5 m long has a uniformly distributed load of w kN per linear metre extending over a length of 3 m from the left-hand reaction. The magnitude of the left-hand reaction is 126 kN and the maximum bending moment occurs at a distance of 2,1 m from this reaction.
Calculate the magnitude of the distributed load w and draw the shearing froce and bending moment diagrams.

Activity 4.3

1. Calculate the co-ordinates of the centroids of the following laminae. [25,9; 22,5]

(a)

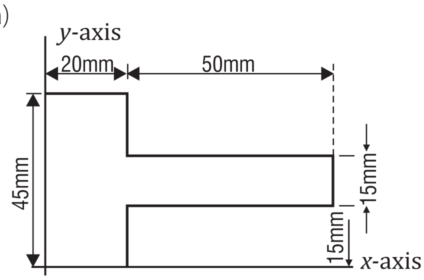


Figure 4.45

(b) A hole, with a diameter of 30mm, is cut out of the rectangular part. [75,86; 40]

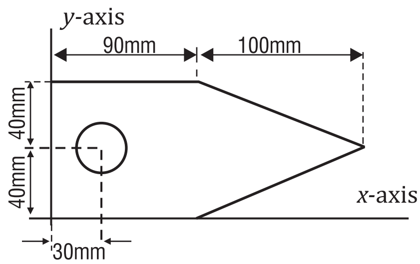


Figure 4.46

(c) [14,24; 15,88]

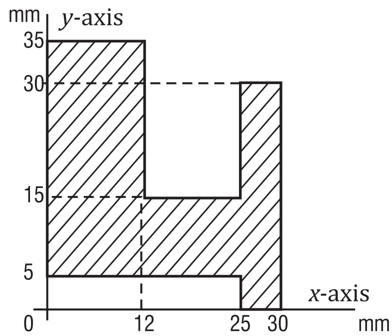


Figure 4.47

(d) [220,45; 175]

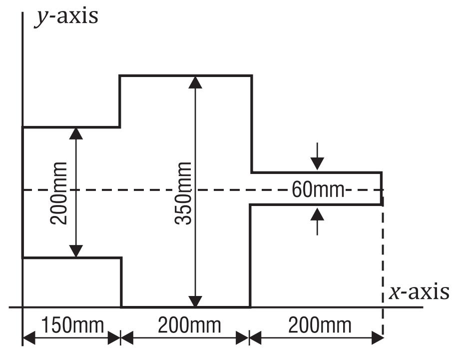


Figure 4.48

Continued overleaf ...

(e) **Figure 4.49** shows a lamina with a hole 150mm in diameter. [281,7;214,68]

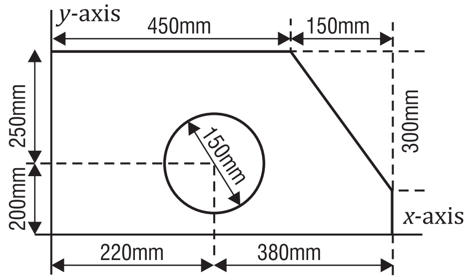


Figure 4.49

(f) [259,99; 150]

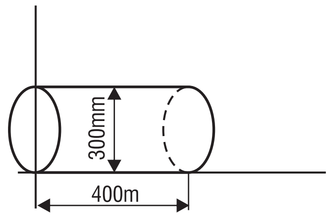


Figure 4.50

2. A pivot of a regulator has a diameter of 80 mm and a total length of 150mm. One side has a conical form and is 90 mm long. Calculate the distance of the centre of gravity from the base. [47,5 mm]

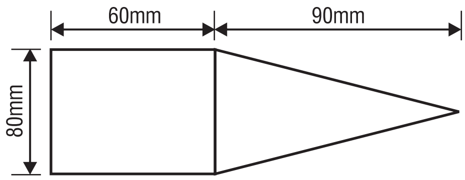


Figure 4.51

3. Calculate the distance of the centre of gravity (centre of mass) from A of a solid cone. [0,6 m]

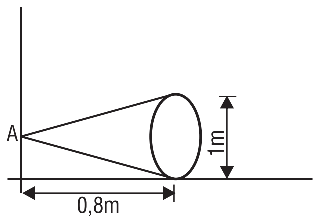


Figure 4.52

4. Calculate the centre of mass of the casting. ($\bar{x}$ = 1 m, $\bar{y}$ = 0,491 m, $\bar{z}$ = 0,5]

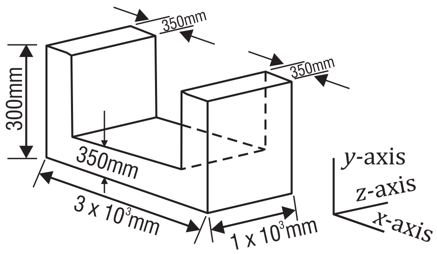


Figure 4.53

5. **Figure 4.54** shows a section of a pneumatic cylinder. Calculate the distance of the centre of gravity from the open side. [171,8mm]

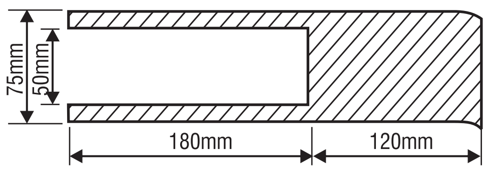


Figure 4.54

6. Calculate the distance of the centre of gravity from point A of the fork link. [176,2 mm]

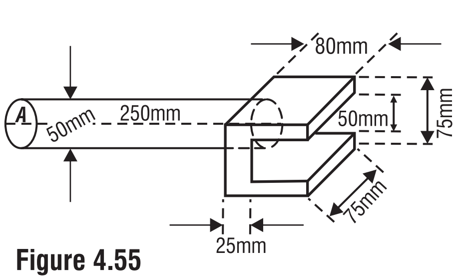


Figure 4.55

7. A steel axle has a diameter of 60 mm and is 360 mm long. Two copper discs are mounted on the axle as shown in **Figure 4.56**. Calculate the distance of the centre of mass from point ***A***. [191,9 mm]

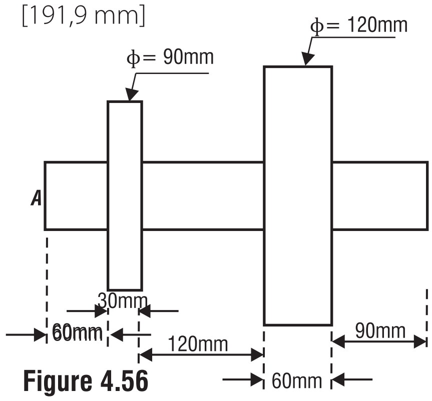


Figure 4.56

Continued overleaf ...

8. Calculate the distance of the centroid of the lined area from the apex. [0,557 m]

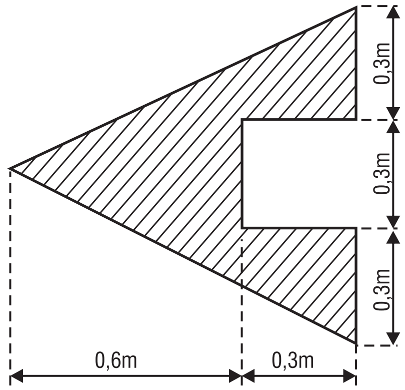


Figure 4.57

9. Calculate the co-ordinates of the centre of gravity of the solid body shown in **Figure 4.58**. [0; 200]

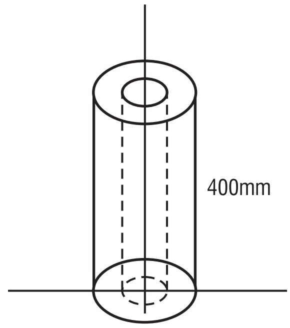


Figure 4.58

Self Check

I am able to:	YES	NO
• Calculate the reactions at the supports of beams subjected to vertical point loads and evenly distributed loads. The mass of the beam is also considered as an evenly distributed load.	○	○
• Draw shear force and bending moment diagrams for simply supported beams subjected to point loads and evenly distributed loads.	○	○
• Determine the positions of the maximum bending moments and their magnitudes.	○	○
• Calculate the centroids of laminae and centres of gravity of solid objects by choosing the moment axis.	○	○

If you have answered "no" to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Module 5

Hydraulics

Learning Outcomes

On the completion of this module the student must be able to:

- Analytically solve practical problems which deal with the determination of:
    - o The volume of liquid required per stroke by the press (slit p be taken into account).
    - o The diameter of the ram of the press or force exerted by the ram.
    - o The pressure in the liquid to overcome a given load.
    - o The work done by the press (efficiency to be taken into account).
    - o The volume of liquid delivered per stroke or in a certain time (slip to be taken into account).
    - o The pressure in the liquid, plunger diameter and force on the plunger.
    - o The rotational frequency of a pump to deliver a given volume of water.
    - o The power required and efficiency with input power given in the case of single, double-and three-cylinder single-acting pumps.
    - o The volume of liquid delivered by the accumulator per working stroke, of the machine it serves or the volume delivered in a given time.
    - o The diameter of the ram and the load required on the accumulator to keep the pressure required constant during the working stroke.
    - o The distance travelled by the accumulator ram during the working stroke.
    - o The transfer of pressure in the liquid between the accumulator and the machine.
    - o The work done and the power with efficiency and slip taken into account.
- Draw sketches which illustrate the working of an accumulator

5.1 Introduction

Hydraulics is a part in applied science and engineering dealing with the mechanical properties of liquids. Fluid mechanics provides the theoretical foundation for hydraulics, which focuses on the engineering uses of fluid properties.

In fluid power, hydraulics is used for the generation, control, and transmission of power by the use of pressurized liquids. Hydraulic topics cover concepts such as pipe flow, dam design, fluidics and fluid control circuitry, pumps, turbines, hydropower, computational fluid dynamics, flow measurement, river channel behavior and erosion.

5.2 Properties of fluids

Fluids are actually incompressible, as the molecules are close together with little space between them. (This is a useful property used in hydraulic machinery such as jacks and lifts.)

A fluid occupies a definite volume irrespective of the shape of the container, while a gas will fill the entire container and therefore does not maintain its volume.

The molecules of fluids and gases slide over one another until stopped by the sides of the container, and thus take up the shape of the container.

Note:
A fluid will seek its own horizontal level.

When a fluid is at rest in a container, the pressure at any point on the same horizontal level must be the same, or the fluid would move (the molecules would slide over one another) until the pressure was at equilibrium.

Fluids exhibit a curved surface (meniscus) where they come in contact with the container.

Definition: cohesion and adhesion

Attraction between molecules of the same type (water molecules and water molecules, for example) is called ***cohesion***, while the attraction between molecules of different substances (water molecules and glass molecules, for example) is known as ***adhesion***.

When water is poured into a narrow glass tube, the edges of the water in contact with the glass will be slightly higher than the fluid surface, because the adhesion of the water molecules to the glass is greater than their cohesion.

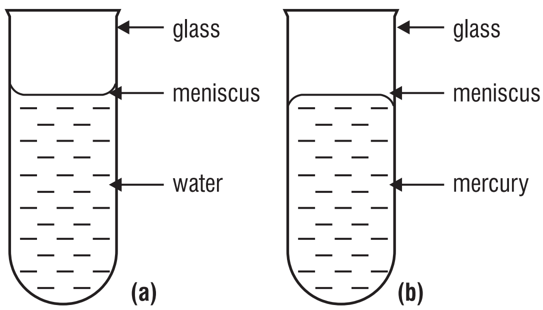


Figure 5.1

With mercury, the adhesion of the molecules to the glass is less than their cohesion, so the mercury surface will be slightly higher than the edges in contact with the glass.

5.2.1 Fluids with enough heat energy form vapour or gas

A gas that loses heat energy condenses to a liquid. An increase in temperature causes the fluid molecules to move faster until some molecules leave the fluid to form gas molecules in the space above the fluid surface.

5.2.2 Fluid surface tension

Molecules that have moved to the surface of a fluid are further apart, so that there is a lateral attraction between them. This creates surface tension. This explain why droplets of fluid are spherical (**Experiment 5.1**).

The force of pressure exerted by a liquid against the sides of its container exists because of the weight of the liquid and the fact that the molecules of the fluid slide easily over one another.

The most important facts relating to pressure exerted by a fluid are the following:

- The pressure exerted by a liquid depends on the density of the liquid - the higher the density the greater will be the pressure exerted (**Experiment 5.2**).
- The pressure in a liquid increases with depth and is directly proportional to the depth (**Experiment 5.3**).
- The pressure at any point in a liquid is exerted in all directions with the same magnitude (**Experiment 5.4**).
- The pressure at any point on the same horizontal plane is the same for a certain liquid (**Experiment 5.5**).
- The pressure is independent of the size and shape of the container (**Experiment 5.6**).
- If an external pressure acts on the surface of a liquid, it is transmitted by the liquid in all directions with the same magnitude (**Experiment 5.7**).

5.3 Simple experiments on the properties of liquids

5.3.1 Experiment 5.1: To show that a fluid surface is under tension

Experiment 5.1

An ordinary sewing needle (or a razor blade) can easily be made to float on water.

Figure 5.2 Section through needle floating on water

- Take a container of pure water and place the needle or blade on a small piece of filter paper, and gently place it on the surface of the water. Within a few seconds, the paper will sink to the bottom and the needle or blade will be left floating.

Close examination reveals that the needle rests in a slight depression. The surface of the water behaves as though covered with an elastic skin. This property is called surface tension.

Even a piece of fine gauze cut and bent into the form of a rectangular container will float. However, if a few drops of alcohol, soap solution or detergent are added to the water the surface tension is decreased and the needle, razor blade or gauze will sink.

5.3.2 Experiment 5.2: To show that the pressure in a liquid Is proportional to the density of the liquid

Experiment 5.2

- Half fill the U-section of a manometer with coloured water. The levels x and y of the coloured water will be the same height for both legs.
- Take measuring cylinders and fill them with different liquids of known densities, such as turpentine, methylated spirits and copper sulphate solution.
- Put one end of the manometer 50 mm deep into the different liquids, one by one. Each liquid will rise in the lower leg to its own, specific height. The coloured water will be higher (at x) in the one leg than (at y) in the other leg.
- In each case take the difference between x and y. This difference represents the pressure at c.

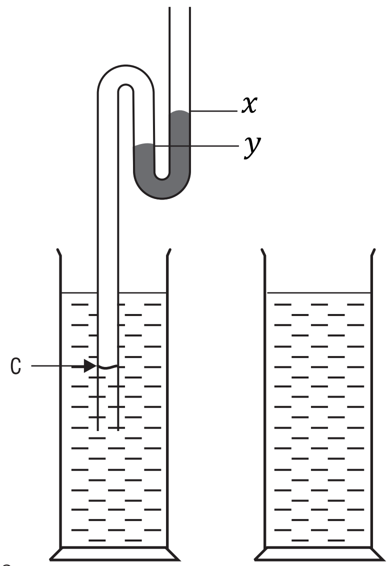


Figure 5.3

The relation between pressure (difference between x and y) and the density of the different liquids will be constant. Therefore, the pressure is directly proportional to the density of the liquid.

5.3.3 Experiment 5.3: To show that pressure in a fluid is directly proportional to depth

Experiment 5.3

- Half fill the U-section of a manometer with coloured water. The water level will be the same for both legs ***x*** and ***y*** because the atmospheric pressure acts on both surfaces.
- Now put the lower end of the manometer a short distance into a glass container with water. The water will rise some distance up the leg while the coloured water will drop in leg ***y*** and rise in leg ***x***, so that levels ***x*** and ***y*** will no longer be the same.
- Measure the distance between the water level in ***c*** and the water surface in the measuring cylinder, and measure the difference between the levels in ***x*** and ***y***.
- Force leg ***c*** further down into the container until level c is twice the previous depth.
- Measure the difference between levels ***x*** and ***y*** again; it will be twice the previous reading.

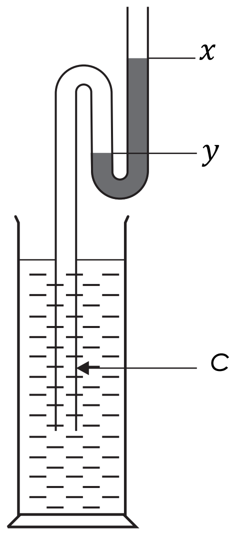


Figure 5.4

Conclusion:
The pressure in a liquid is directly proportional to the depth.

5.3.4 Experiment 5.4: To show that the pressure In the liquid remains constant at the same horizontal plane

Experiment 5.4

- Fill the U-section of a manometer with coloured water and put the lower end into a cylinder filled with water. The water will rise a distance up the lower leg, to **c**.
- The fluid levels of the coloured water in the manometer will no longer be the same.
- Keep the depth of the lower end of the manometer constant below the fluid surface and move it to other positions.
- Take readings at other depths.

The observations may be repeated for different liquids such as turpentine and methylated spirits.

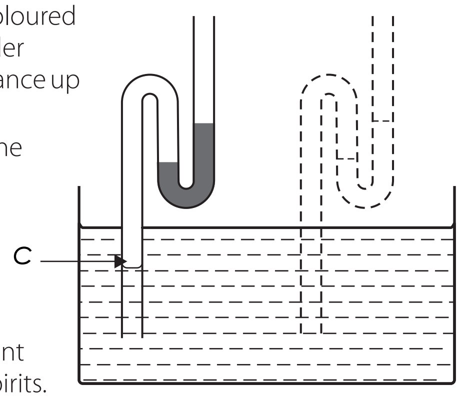


Figure 5.5

Result:
If the depth of the manometer is kept constant, the difference between the heights of the columns of coloured water is constant.

Conclusion:
The pressure in a liquid at the same depth (horizontal plane) is constant.

5.3.5 Experiment 5.5: To show that the pressure at a point In a liquid is the same in all directions

Experiment 5.5

- Fill the U-section of three differently shaped manometers with coloured liquid until the liquid levels in all three are at the same height.
- Put the manometers into a bucket filled with water so that the openings of all three are at the same height.

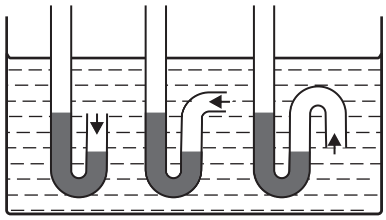


Figure 5.6

Result:

The difference between the heights of the fluid levels in all three manometers is the same.

Conclusion:

At a certain height the pressure in any direction (upwards, downwards and sideways) is the same.

5.3.6 Experiment 5.6: To show that the pressure Is Independent of the size or shape of the container

Experiment 5.6

When fluid is poured into a special container, the fluid level in each separate section of the container will be the same.

- Take a manometer and put the lower end in each of the separate sections of the container but each time to the same depth.
- The difference in height of the fluid surfaces (pressure in a liquid) of the coloured water in the manometer will be the same.
- Repeat the observations at different depths.
- The pressure will again be constant.

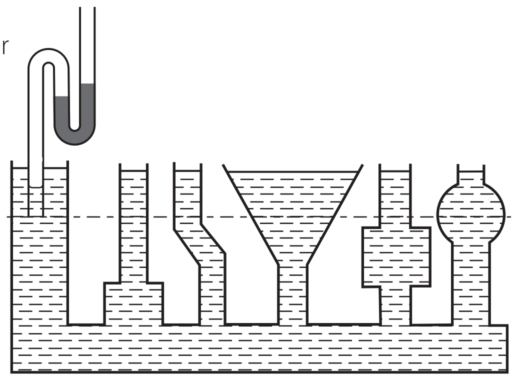


Figure 5.7

Conclusion:

The pressure in the container is the same at any point at the same horizontal plane; if it were not, the fluid would have moved until the pressure was uniform.

The fact that the fluid is at the same height in each separate section, irrespective of the shape of the container, means that the pressure at any point in the liquid will change only with vertical depth, irrespective of the shape of the container.

5.3.7 Experiment 5.7: To show that an external pressure exerted on a fluid is transmitted in all directions through the liquid with the same intensity

Experiment 5.7

- Fill the special flask with water. The flask has several evenly spaced holes of equal size. The water does not flow from the holes with equal strength, but it flows more strongly from the lower holes.
- Repeat the experiment, but use the special piston in the upper part of the flask: force it down. The water will squirt in all directions from the holes, but with equal force.

Conclusion:
The pressure acts with the same magnitude in all directions within the fluid.

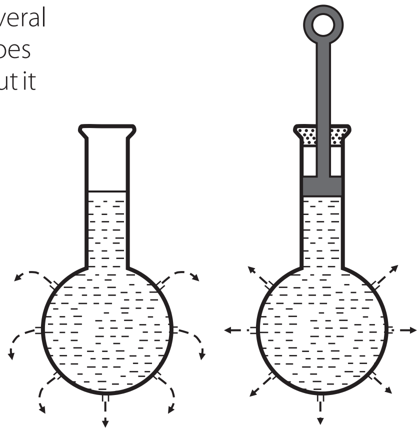


Figure 5.8

Note:
Experiment 5.7 is also used to prove Pascal's law.

5.4 Pascal's law

The pressure exerted upon a fluid surface acts with the same magnitude in all directions.

Definition: Pascal
The Pascal is the pressure that arises when a force of 1 N is applied over and perpendicular to an area of 1 m^2.

5.5 Pressure

Pressure is force per unit area. $\therefore$ Pressure (p) $= \dfrac{applied\ perpendicular\ to\ the\ area}{area\ (A) over\ which\ the\ force\ is\ applied}$

$$\therefore p = \frac{F}{A}$$

There is a difference between the term ***pressure*** and ***force of pressure*** (thrust).

Pressure (p) = force per unit area in pascals (Pa).
Force of pressure (F) = total force over whole area (N).

$\therefore p = \dfrac{F}{A}$ In words: **force of pressure = pressure x area.**

or $F = pA$

5.6 Pressure in liquids (hydrostatic pressure)

Consider a certain area A (**Figure 4.9**) at depth h (height of a fluid column) below the fluid surface.

The mass of the vertical fluid column causes a downwards force acting perpendicular to the area. This force is the weight of the fluid column, and is distributed evenly over the area.

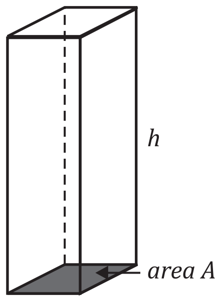


Figure 5.9

Continued overleaf ...

The mass of the fluid column depends on the volume V and the density of the fluid where:

$$Density\ \varrho = \frac{mass\ m}{volume\ V}$$

Density of pure water at 4 °C = 1 000 kg/m^3

Relative density of a substance is the ratio of the density of the substance to the density of pure water, which is used as a standard reference.

$$Relative\ density\ of\ a\ substance = \frac{density\ of\ substance}{density\ of\ water}$$

5.6.1 Pressure at a point In a liquid owing to depth

Let the area ***A*** in **Figure 5.9** represent a point at a depth h in a liquid of density ϱ.

According to the definition of pressure, it follows that: ***Pressure p at a point area A*** = $\frac{Force\ F}{Area\ A}$

$$\therefore p = \frac{F}{A}$$

$$\therefore p = \frac{\varrho\ V\ g}{A} \qquad \begin{pmatrix} with\ F = weight \\ = mg \\ where\ m = \varrho\ V \\ \therefore F = \varrho\ V\ g \end{pmatrix}$$

$$\therefore p = \frac{\varrho\ A\ h\ g}{A}$$

$$\therefore p = \varrho\ h\ g$$

In words: ***Pressure at a point = density x height x gravitational acceleration***.

5.6.2 Force of pressure exerted on an area by a vertical column of liquid

The pressure exerted on an area by a vertical column of liquid is the force or weight acting on the area. According to the definition of pressure it follows that:

Force of pressure $F = pressure\ p\ x\ area\ A$

$\therefore F = p\ A$

$= \varrho hgA\ as\ p = \varrho hg$

In other words:

force of pressure on an area = density x height x gravitational acceleration x area.

It is therefore obvious that the force of pressure will increase if the depth or density increases.

5.6.3 The pressure on a fluid surface

According to Pascal's law, the pressure on a confined fluid surface is transmitted in all directions with the same intensity. If the pressure on the fluid surface (atmospheric pressure) is also taken into account, the pressure at a point also increases and, therefore, so does the force of pressure on the point.

5.6.4 Total pressure at a point in a liquid

Let Po be the pressure on the surface of a liquid and p the pressure at a point at a depth h below the liquid surface. Then the total pressure PT at the point is:

$$PT = P_o + P = P_o + \varrho gh\ as\ p = \varrho gh$$

5.6.5 Total force of pressure exerted on an area by a vertical fluid column and an external pressure

According to the definition of pressure it follows that:

$F = pA$

$\therefore Total\ force\ of\ pressure\ F_T = total\ pressure$

$P_T \times area$

$F_T = p_T A$

$= (p_o + \varrho hg)A$

5.6.6 Gauge pressure

Gauge pressure is the pressure registered on a gauge. The gauge does register atmospheric pressure but is calibrated to register it as zero.

If a gauge on a container indicates a pressure of 200 kPa, the atmospheric pressure is not included in that figure.

5.6.7 Absolute pressure

The absolute pressure equals the gauge pressure plus the atmospheric pressure.

Worked Example 5.1

Calculate the pressure at a depth of 200 m below sea level if the density of sea water is 1 030 kg/m^3. Ignore atmospheric pressure. Take gravitational acceleration as 10 m/s^2.

Solution:

Given: $h = 200\text{m}; \varrho = 1\ 030\text{kg/m}^3; g = 10\ \text{m/s}^2$

Pressure $p = \varrho gh$

$p = 1\ 030 \times 10 \times 200$

$= 2{,}06\ \text{MPa}$

Worked Example 5.2

Calculate the force of pressure exerted on a circular plate, 0,25 min diameter, that lies on the bottom of a swimming pool. The plate is 2,5m below the surface of the water and the atmospheric pressure is 101,3kPa. Take g = 10m/s^2.

Solution:

Given: $d = 0{,}25\ \text{m};\ h = 2{,}5\ \text{m};\ P_o = 101{,}3\ \text{kPa};\ g = 10\ \text{m/s}^2$

Force of pressure $F = (P_o + \varrho gh)A$

$= (101{,}3\ x\ 103\ + 1\ 000\ x\ 10\ x\ 2{,}5)^{\pi}/4(0{,}25)^2$

$= 6{,}20\text{kN}$

Worked Example 5.3

Calculate the force of pressure exerted on the bottom of a rectangular water container 2 m long and 1,5 m wide if it is filled with water to a height of 1,7m. The container is sealed off and the water surface is under a pressure of 200kPa . $g = 10\ \text{m/s}^2$.

Solution:

Given: length of container = 2 m; width of container = 1,5 m; height = 1,7 m; Po = 200 kPa; g = 10 m/s^2.

Force of pressure $F = (P_o + \varrho gh)A$

$= (200 \times 10^3 + 1\ 000 \times 10 \times 1,\ 7)2 \times 1{,}5$

$= 65{,}1\ \text{kN}$

5.7 Transmission of fluid pressure

The hydraulic press (**Figure 5.10**) is a practical example of Pascal's law; the transmission of pressure in a liquid is clearly illustrated.

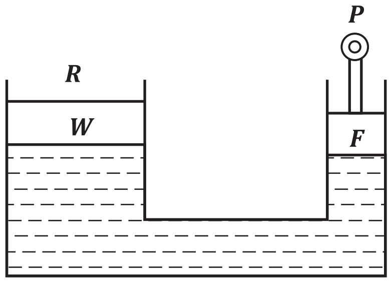


Figure 5.10

5.7.1 Relation between applied force and load lifted

According to Pascal's law, the fluid pressure caused by the force ***F*** below piston ***P*** is transmitted to the lower end of piston ***R*** in order to support ***W*** (**Figure 5.10**).

Pressure below piston ***P*** **=** pressure below piston ***R***

$$\therefore \frac{F}{A_1} = \frac{W}{A_2}$$

$$\frac{F}{\pi/4d^2} = \frac{W}{\pi/4D^2}$$

$$\frac{F}{d^2} = \frac{W}{D^2}$$

$$\text{or } \frac{W}{F} = \frac{D^2}{d^2}$$

A_1 = area of P
A_2 = area of ***R***
W = load supported by ram piston ***R***
F = force applied to plunger ***P***
d = diameter of plunger
D = diameter of ram piston

5.7.2 Relation between load on plunger and effort on lever

The effective force on the plunger is greater than the force (effort) applied to the handle of the lever.

$$\therefore \text{ Mechanical advantage MA } = \frac{load\ on\ plunger\ F}{effort\ on\ handle\ W}$$

The derivation of the above formulae of hydraulic presses assumes ideal conditions; no provision is made for losses due to friction or fluid leaks.

5.7.3 Efficiency

The efficiency of hydraulic presses may be calculated as follows:

$$efficiency\ \eta = \frac{work\ output}{work\ input} \times 100\%$$

5.7.4 Relation between plunger movement and ram-piston movement

Work done by plunger = work done by ram
Force on plunger x distance moved by plunger
= force on ram x distance moved by ram
∴ F x plunger distance = W x ram distance

Also:

Volume of fluid displaced by plunger = volume of fluid that displaces ram.

$$\therefore \frac{\pi}{4}d^2 \text{ x stroke length x number of strokes}$$

$$= \therefore \frac{\pi}{4}D^2 \text{x distance moved by ram}$$

Worked Example 5.4

The ram diameter of a hydraulic press is 480 mm. It exerts a force of 15 kN on the load if a force of 320 N is applied to the plunger.

Calculate:

(a) the fluid pressure
(b) the plunger diameter

Solution:

Given: D = 480 mm; W = 15 kN; F = 320 N

(a) pressure p = $\frac{force\ F}{area\ A}$

$\therefore p = \frac{15 \times 10^3}{\pi/4(0{,}48)^2}$

$= 82{,}89$ kPa

(b) According to Pascal's law, the pressure is transmitted in all directions. The pressure below the plunger and ram is the same, therefore,

$p = \frac{F}{A}$

$\therefore 82{,}89 \times 10^3 = \frac{320}{\pi/4d^2}$

$\therefore d^2 = 4{,}915 \times 10^{-3}$

$= 0{,}070$ m

$= 70$ mm

Worked Example 5.5

The following data refer to a single-acting hydraulic press:

Plunger diameter	= 15 mm
Plunger stroke	= 12 mm
Force applied to plunger	= 300 N
Diameter of ram	= 55 mm

If the mechanical efficiency of the hydraulic system is 80 per cent and that of the lever system is 85 per cent, calculate:

(a) the volume of liquid displaced after five pumping strokes of the plunger
(b) the distance in mm that the ram will be moved after five pumping strokes
(c) the force exerted by the ram
(d) the effort that must be applied to the lever in order to obtain a force of 300N on the plunger if the mechanical advantage of the lever system is 10
(e) the mechanical advantage of the hydraulic system.

Solution:

Given: d = 15 mm; D = 55 mm; F = 300 N; η lever = 85 %;
η hydraulic system = 80 %; stoke length of plunger = 12 mm;
number of strokes = 5

(a) Volume of liquid displaced by plunger

$= \pi/4d^2 \times stroke\ length \times number\ of\ strokes$

$= \pi/4(0{,}015)^2 \times 0{,}012 \times 5$

$= 10{,}603 \times 10^{-6}\ m^3$

(b) Volume displaced by plunger = volume required by ram

$\therefore 10{,}603 \times 10^{-6} = \pi/4(0{,}055)^2 \times h$

$\therefore h = 4{,}463 \times 10^{-3}$ m

$= 4{,}463$ mm

(c) $\frac{W}{F} = \frac{D^2}{d^2}$

$\therefore \frac{W}{300} = \frac{0{,}055^2}{0{,}015^2}$

Continued overleaf ...

W = 4,033 kN for an ideal press

$\eta = \frac{actual\ force\ on\ ram}{applied\ force\ on\ ram} \times 100$

$\therefore 80 = \frac{actual\ force\ on\ ram}{4{,}033} \times 100$

$\therefore$ Actual force on ram $= \frac{80 \times 4{,}033}{100}$

$= 3{,}226$ kN

(d) $MA = \frac{load\ F}{effort}$

$10 = \frac{300}{effort}$

effort $= \frac{300}{10}$

$= 30$ N

This is the actual effort required to produce a load of 300 N under ideal conditions.

$\therefore \eta = \frac{actual\ effort}{applied\ effort} \times 100$

$\therefore 85 = \frac{30}{applied\ effort\ required} \times 100$

$\therefore$ applied effort required

$= \frac{30 \times 100}{85}$

$= 35{,}294$ kN

(e) MA for hydraulic systems

$MA = \frac{load\ at\ ram}{effort\ at\ plunger}$

$= \frac{W}{F}$

$= \frac{3\ 226}{300}$

$= 10{,}743$

Worked Example 5.6

The ram of a hydraulic lift produces a force of 24 kN when a force of 400 N is applied to the plunger. The plunger diameter Is 80 mm.

Calculate:

(a) the diameter of the ram

(b) the fluid pressure.

Solution:

Given: W = 24 kN; F = 400 N; d = 80 mm

(a) $\frac{W}{F} = \frac{D^2}{d^2}$

$\frac{24 \times 10^3}{400} = \frac{D^2}{80^2}$

$D^2 = 384\ 000$

$D = 619{,}677$ mm

(b) $p = \frac{F}{A}$

$= \frac{400}{\pi/4(0{,}08)^2}$

$= 79{,}577$ kPa

5.8 Important formulae for pressure in a liquid

- Pressure $p = \frac{force\ (F)}{area\ (A)}$ $\quad p = \frac{F}{A}$
- Density (ϱ) $= \frac{mass\ (m)}{volume\ (V)}$ $\quad \varrho = \frac{m}{V}$
- Pressure (p) at a point = *density* (ϱ) *x gravitational acceleration* (g) *x height* (h) $p = \varrho g h$
- Force of pressure (F) due to a fluid column = *pressure* (p) *x area* (A) $F = \varrho g h A$
- Total pressure (P_T) on a point in a liquid = *pressure on surface* (P_o) + *pressure* (p) at a point in the fluid. $\quad p_T = p_o + p$

Continued overleaf ...

- Total force of pressure (FT) on a point in a fluid = total pressure (p_T) at the point x area (A)
  $F_T = p_T \times A$
  $(po + \varrho gh)A$
- Absolute pressure = gauge pressure + atmospheric pressure
- Pressure below plunger = pressure below ram
  $\frac{W}{F} = \frac{D^2}{d^2}$
- Mechanical advantage = $\frac{load}{effort}$
- Efficiency $\eta = \frac{work\ output}{work\ input} \times 100$
- F x plunger distance = W x ram distance
- Volume displaced by plunger = volume required by ram:
  d^2 x stroke length x number of strokes
  = D^2 x distance moved by ram

5.9 Work done by a liquid

The amount of work that may be done by a liquid depends on the work needed to transmit the liquid vertically, horizontally or against pressure. The volume of liquid transmitted has weight due to gravity, and it must be transmitted over a certain distance or to a certain height.

When a force is moved over a distance, work is done. To deliver liquid at a pressure, centrifugal pumps, piston pumps, or potential energy (which is available due to the relative height of the liquid) may be used.

5.9.1 Work done by a liquid due to height

Any liquid that is available at a higher level than that of the outlet can do work. Water in dams and reservoirs may be used to do work by driving turbines or water mills. These devices may then be used to drive generators in hydroelectric power stations and machines.

The work done, may be calculated as follows:

$$Work\ done\ (WD) = potential\ energy\ (Ep)$$
$$WD = (mg)h$$
$$= force\ (F)\ x\ distance\ moved\ (s)$$
$$= Fs$$

Worked Example 5.7

Calculate the work that may be done by water in a dam that is 100 m above the turbine of a power station. The water is fed to the turbine through a pipe of I m internal diameter. Take the density of the water as 1 000 kg/ms and gravitational acceleration as 10 m/s².

Solution:

Given: h = 100 m; d = 1m

ϱ = 1000 kg/m³; g = 10 m/s²

$\varrho = \frac{m}{V}$

$m = \varrho V$

= 1 000 x $\pi/4$ x 1^2 x 100

= 78,54 kg

Mass of water is 78,54 kg

WD = Ep

= mgh

= 78,54 x 10^6 x 10 x 100

= 78,54 GJ

5.9.2 Work done to deliver water at pressure

There are many practical examples of the need to deliver water against pressure: irrigation for agriculture, the spraying of herbicides the delivery of water to reservoirs of cities and so on.

If all losses are neglected, it follows that for a piston pump:

Work done = *force x distance moved*
= *(pressure x area) x distance moved*
= *pressure x area x stroke length*
= *pressure x volume*
WD = *pV*

Worked Example 5.8

A water pump, in which the piston diameter is 150 mm and the stroke length is 250 mm, operates at a fluid pressure of 450 kPa.

Calculate:

(a) the volume water per working stroke
(b) the force exerted by the piston
(c) the work done per operating stroke.

Solution:

Given: diameter= 150 mm; stroke length = 250 mm; pressure 450 = kPa

(a) Volume (V) per stroke length
$= \pi/4(0{,}150)^2 \times 0{,}250$
$V = 4{,}418 \times 10\text{-}^3\ m^3$

(b) Force (F) exerted by piston
= pressure (p) x area (A)
F = pA
$= 450 \times 10^3 \times \pi/4(0{,}150)^2$
= 7,952 kN

(c) Work done (WD) per working stroke
= force (F) x distance moved
WD = F x stroke length
$= 7{,}952 \times 10^3 \times 0{,}250$
= 1988,48 J

Worked Example 5.9

The work done during the working stroke of a water pump is 4,2 kJ. The effective force on the piston is 20 kN and the piston diameter is 245 mm.

Calculate:

(a) the water pressure during the working stroke
(b) the stroke length of the pump
(c) the volume of water delivered per working stroke
(d) the mass of water delivered per workmg stroke

Solution:

Given: WD = 4,2 kJ; F = 20 kN; d = 245 mm

(a) $p = \frac{F}{A}$
$= \frac{20 \times 10^3}{\pi/4(0{,}245)^2}$
= 424, 236 kPa

(b) WD = force x stroke length $4{,}2 \times 10^3$
= 20 x 103 x stroke length stroke length
= 210 mm

Continued overleaf ...

(c) WD = p.V

$4{,}2 \times 10^3 = 424{,}236 \times 10^3$ V

V = 9,9 x 10-3 m^3

= 9,9l

(d) $\varrho = \frac{m}{V}$

$1\,000 = \frac{m}{9{,}9 \times 10^{-3}}$

m = 9,9 kg

5.9.3 Work done to deliver water at a head

Static head is measured from the water level up to the height to which the water must be lifted (delivered).

Static head = suction head + delivery head.

The static head is the effective distance through which a certain volume of water must be delivered above the water level.

The volume of water in the pipe that is higher than the water level has a certain mass and therefore has weight due to gravity.

If losses are neglected it follows that:

Work done (WD) = force (F) x distance moved (h)

WD = mass (m) x gravitational acceleration (g) x head (h)

= mgh

Compare this work done with the work that could have been done by a liquid due to its height.

Also: *WD = mgh where* $\varrho = \frac{m}{V}$

= (Vϱ*)gh*

Worked Example 5.10

The inner diameter of a water pipe is 55 mm and the effective head is 35 m. Calculate the work done to deliver water to this height if the density of the water is 1 000 kg/m^3 and g = 10 m/s^2.

Solution:

Given: d = 55 mm; h = 35 m; ϱ = 1 000 kg/m^3

WD = Vϱ gh

= $\pi/4(0{,}055)^2$ x 35 x 1 000 x 10 x 35

= 29,104 kJ

Worked Example 5.11

The water level in a borehole is 30 m from the ground surface. Water is pumped into a reservoir that is 7 m above the ground surface.

Calculate:

(a) the work done to pump water into the reservoir if both the suction and the delivery pipes have a diameter of 50 mm

(b) the reading on a pressure gauge in the delivery pipe, 1 m above the ground. Take for water as 1 000 kg/ m^2.

Solution:

Given: Suction head = 30m; delivery head = 7m; d = 50mm; ϱ = 1 000kg/m^3

(a) Effective head = suction head + delivery head

= 30 + 7

= 37m

WD = mgh

= Vϱgh

= π/4(0,05)2 x 37 x 1 000 x 10 x 37

= 26,88 kJ

(b) p = ϱgh

= 1000 x 10 x 6

= 60 kPa

5.10 Important formulae for work done by a liquid

- Work done = force x distance moved
  $WD = Fxs$
- Work done = pressure x volume
  $WD = pxV$
- Work done = potential energy
  $WD = mgh$
- Effective head = suction head + delivery head.

5.11 The hydraulic accumulator

Hydraulic systems are usually fitted with accumulators to reduce the plant and operating cost of large pumps that are only in operation at intervals. The operating stroke of a hydraulic press takes only a few seconds, after which the object that has been worked on is removed.

This means that the fluid which is delivered under continuous pressure is utilised for only a few seconds.

The accumulator is a device to store the unutilised high pressure during the time when the press is not in operation, and to deliver it to the press when needed. It is thus unnecessary to switch the supply on and off to ensure that the supply will be delivered at a constant pressure.

Energy is therefore stored in the accumulator when the delivery pumps supply fluid at a higher rate than needed by the press and released to increase the supply when the direct pump delivery is insufficient.

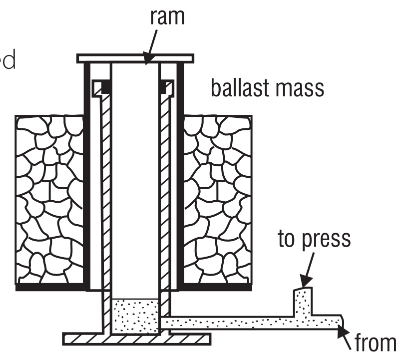


Figure 5.11

Figure 5.11 shows a sketch of a simple accumulator. It consists of a ram working in a vertical cylinder. When fluid is delivered faster than needed, the ram rises to store energy.

As soon as the demand for fluid exceeds the capacity of the supply pump the ram descends.

When the ram has reached its maximum height, a valve cuts off the supply and when the ram drops the supply is switched on again.

Assume the accumulator with ballast mass weighs W newton, the ram diameter is D millimetre and that the fluid pressure in the accumulator is p Pascal, it follows that for equilibrium:

$$W = p\frac{\pi}{4}D^2 \qquad \therefore p = \frac{4W}{\pi D^2}$$

As W and D are constants it follows that the pressure (p) remains constant when an accumulator is fitted to a hydraulic system.

The amount of energy that can be stored in an accumulator is equal to the work done in order to lift the ram.

Energy = work done
E = W x h (h = height that the ram has been lifted)

Worked Example 5.12

The diameter of the ram of an accumulator is 200 mm. Calculate the mass of the ballast which is needed to obtain a pressure of 160 kPa.

Solution:

Given: D = 200 mm; p = 160 kPa

$$Weight\ W = p\frac{\pi}{4}D^2$$
$$= 160 \text{ x } 10^3 \text{ x } \frac{\pi}{4} \text{ x } 0{,}2^2$$
$$= 5026{,}95 \text{ N}$$
$$\therefore Mass\ m = \frac{5026{,}55}{10}$$
$$= 502{,}655 \text{ kg}$$

Worked Example 5.13

An accumulator with a ram diameter of 250 mm is used in conjunction with a press and a three-stroke pump. The press must operate at one operating stroke per minute while it uses 0,1 m^3 water.

The operating stroke takes 6 seconds. Calculate the smallest permissible rise of the ram if the plunger of the pump has a diameter of 70 mm and a stroke length of 250 mm. Also calculate the speed at which the pump operates if there is a slip of 5%.

Solution:

The pump must deliver 0,1 m^3 water per minute. It will deliver 0,01 m^3 during 6 seconds and the accumulator will deliver 0,1 - 0,01 = 0,09 m^3 of water.

Volume of water needed for a minimum rise in height of h metres:

$$V = \frac{\pi}{4}D^2h$$
$$0{,}09 = \frac{\pi}{4}(0{,}250)^2h$$
$$\therefore h = \frac{0{,}09\ x\ 4}{\pi \times (0{,}250)^2}$$
$$= 1{,}833 \text{ m}$$

The stroke volume of the pump is:

$$V = \frac{\pi}{4}d^2h$$
$$= \frac{\pi}{4}(0{,}07)^2 \times 0{,}25$$
$$= 0{,}000962 \text{ m}^2$$

Number of operating strokes with a slip of 5%:

$$= \frac{0{,}1}{0{,}95 \text{ x } 0{,}000962}$$
$$= 109{,}42$$

The pump has three plungers:

$$\therefore \text{ Speed of pump } = \frac{109{,}42}{3}$$
$$= 36{,}5 \text{ r/min}$$

Activity 5.1

Take g = 10 m/s^2 and the density of pure water as 1 000 kg/m^3.

1.
(a) Why do liquids take the form of the container into which they are poured?
(b) Name five properties of fluids that will distinguish them from solids.
(c) Describe, with the aid of a neat sketch, an experiment to indicate that the pressure at a point in a liquid is independent of the shape and size of the container.

2.
(a) Explain the difference between pressure in a liquid and force of pressure (thrust) on a point.
(b) A plate that measures 0,5 m square lies on the bottom of a swimming pool 2, 7 m below the water level.

Calculate:

(i) the pressure at the bottom of the swimming pool [27 kPa]
(ii) the thrust on the plate [6,75 kN]
(iii) the thrust on the bottom of the swimming pool if it has the same depth throughout and is 4 m wide and 6 m long. [648 kN]

3.
(a) What factors influence the pressure at a point in a liquid?
(b) Describe a simple experiment to show that fluid pressure is directly proportional to the density of the fluid.
(c) Calculate the pressure at a point 0,5 m below the fluid level in a container if it is filled with:
(i) water with a density of 1 000 kg/m^3 [5 kPa]
(ii) milk with a density of 1 032 kg/m^3 [5,160 kPa]
(d) Calculate the pressure at the bottom of a container if it is 1 m deep and filled with water. [10 kPa]

4.
(a) Will the pressure on the surface of a liquid have any effect on the pressure at a point in a liquid?
(b) Describe a simple experiment to show that fluid pressure is directly proportional to the depth of a liquid.
(c) A closed cylinder 100 mm long with a radius of 20 mm is immersed vertically into a liquid of relative density 1,3. The upper edge of the cylinder is 150 mm below the liquid surface.

Calculate:

(i) the pressure on the upper face of the cylinder [1,95 kPa]
(ii) the pressure on the lower face of the cylinder [325 kPa]
(iii) the force of pressure on the upper face of the cylinder if the atmospheric pressure is neglected [2,45 N]
(iv) the force of pressure on the lower face of the cylinder if the atmospheric pressure is 101,3 kPa [131,38 NJ]
(v) the pressure at a point 2 m below the fluid surface if the atmospheric pressure is neglected. [26 kPa]

5.
(a) Explain the term "propagation of pressure".
(b) The force on the master cylinder of the brake mechanism of a car is 300N when the pedal is depressed. The master cylinder is connected hydraulically to the wheel cylinders. Calculate the force exerted by the wheel cylinders on the brake shoes when the diameter of the master cylinder is 19 mm and the diameter of the wheel cylinders is 30 mm.
Also, calculate the pressure in the brake system. [747,92 N; 1,058 MPa]

Continued overleaf ...

6.
(a) Define Pascal's law.
(b) Pascal's law is clearly illustrated by the hydraulic press. In a hydraulic press with a ram diameter of 560 mm, a force of 250 N is applied to the plunger, which is of 80 mm diameter.

Calculate:
(i) the fluid pressure [49,736 kPa]
(ii) the weight of the load on the ram. [12,25 kN]

7.
(a) Describe a small experiment to show that pressure is independent of the magnitude or shape of the container.
(b) Calculate the thrust on the bottom of each of the containers in **Figure 5.12**. Each container has a circular bottom and is filled with water to a depth of 0,3 m. [23,56N]

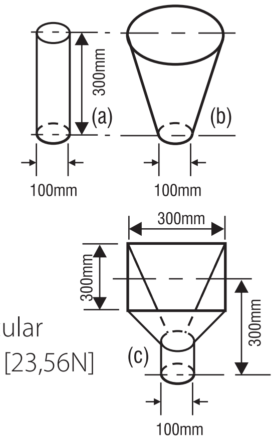


Figure 5.12

8.
(a) Describe a simple experiment to show that a. fluid surface will always be under tension.
(b) A circular water container has a height of 1,5 m and a diameter of 0,5m. It is mounted vertically with a vertical outlet pipe fitted to the upper surface. The pipe has a cross-sectional area of 100 cm^2. The container is filled with water until the water level in the pipe reaches a height of 1,2 m.

Calculate:
(i) the thrust on the bottom of the container [5,301 kN]
(ii) the thrust on the inside of the section of the container to which the pipe is fitted. Ignore atmospheric pressure. [2,236 kN]

9. Calculate the pressure required for a water-supply system that will raise water 25 m vertically. [250 kPa]
10. A rectangular container 300 mm by 450 mm has a height of 500 mm. It is exactly half-filled with water. Calculate the force of pressure on the bottom of the container if it is sealed off and filled with air at a pressure of 120 kPa. [16,538 kN]
11. The area of the piston of a pressure pump is 50 cm^2. Calculate the force that must be applied to deliver water to a height of 30m. [1,5 kN]
12. Calculate the force exerted by a ram piston if an effort of 130 N is applied to the end of the lever of the hydraulic press. The mechanical advantage of the lever system is 12 and the efficiency of the press is 80 per cent. The diameter of the plunger is 20 mm and that of the ram is 200 mm. [124,8 kN]
13. The diameter of the ram of a hydraulic jack 100 mm. The plunger has a diameter of 25 mm and a stroke of 50 mm. The mechanical advantage of the lever is 15.

Calculate:
(a) the force that must be applied to the lever to lift a load of 2 Mg If the efficiency is 86 per cent [96,9 kN]
(b) the number of strokes for the lever to lift the load by 150 mm [48]
(c) the number of strokes needed to lift the load 182,2 mm if there is a slip of 6 percent. [62]

14. The force exerted on the ram piston of a hydraulic press is 5,2 kN if a force of 260 N is exerted on the plunger. The diameter of the plunger is 80 mm.

Calculate:
(a) the diameter of the ram [357,8 mm]
(b) the mechanical advantage of the hydraulic system. [20]
(c) the amount by which the ram will be raised if the plunger makes 40 strokes raise [160 mm]
(d) the efficiency of the press if the mechanical advantage is 13. [65%]

15. In a hydraulic brake system, a force of 250 N is applied to the brake pedal, which has a mechanical advantage of 6:1. The diameter of the brake master cylinder against which the resultant force of the brake pedal acts is 20 mm, while the diameter of each of the wheel cylinders upon which the pressure of the master cylinder acts is 30 mm. The efficiency of the brake system is 75 per cent.

Calculate:

(a) the total force exerted by the wheel cylinders on the brake shoes [20,25kN]

(b) the distance moved by the brake pedal lever to take up the original free space of 0,5 mm between the brake drum and brake shoes. [54mm] NB: Each of the eight wheel cylinders presses against a brake shoe having a clearance of 0,5 mm.

Activity 5.2

Take the density of water as 1 000 kg/m^3 and g = 10 m.s-^{2}.

1.

(a) Water is pumped into a dam through a delivery pipe of which the outlet is 60 m above the pump. 800 kJ energy Is needed to pump water into the dam. Calculate the amount of work that could be done if the water in the dam were used to drive a centrifugal pump, 60 m below the outlet of the dam. [800 kJ]

(b) Calculate the pressure in the delivery pipe at the pump that is 60 m below the delivery pipe outlet. [600 kPa]

2.

(a) Calculate the work done to deliver 3 m^3 water at a height of 40 m. [1,2MJ]

(b) Calculate the work done to deliver 5 m^3 water at a pressure of 200 kPa. [1 MJ]

(c) Calculate the work done to transport water of mass 500 kg horizontally over a distance of 1 km. [5 MJ]

(d) During the operation stroke, the effective pressure in a single-action piston pump Is 500 kPa. The piston diameter is 120 mm and the stroke length is 220 mm.

Calculate:

(i) the volume of water delivered per stroke [2,488 x 10^{-3}m^3]

(ii) the work done per stroke. [1,244 kJ]

3.

(a) Calculate the work done during 1 min by the water from a waterfall delivering 200 l a minute if the water falls through 10 m. [20 kJ]

(b) Calculate the power that could be delivered by the waterfall. [333,3 W]

4. A water pump with a piston diameter of 160 mm must deliver 5 x 10-3 m^3 water per operating stroke. The force on the piston is 9,6 kN.

Calculate:

(a) the stroke length [248,7 mm]

(b) the work done per operating stroke [2,387 kJ]

(c) the liquid pressure per operating stroke [477,5 kPa]

(d) the mass of the water delivered per operating stroke [5 kg]

5.

(a) Calculate the work done in pumping all the water from a circular dam with a diameter of 15 m if the pressure at which water is forced through the irrigation sprinklers is 400 Pa. The depth of the water in the dam is 2m. [141,37 kJ]

(b) Calculate the power required to empty the dam in 10 hours. [3,93 W]

6. A water pump delivers 10 kg water per operating stroke if the effective force on the piston is 18 kN. The piston's diameter is 230 mm.

Calculate:

(a) the volume water in m^3 delivered per operating stroke [0,01 m^3]

(b) the stroke length of the piston [240,69 mm]

(c) the average water pressure [433,24 kPa]

(d) the work done per operating stroke. [4 332,4 J]

7.

(a) An irrigation system operates at a pressure of 80 kPa and the pump itself is 30 m lower than the sprinklers The pipes of the system have an inner diameter of 32 mm. Calculate the work done in pumping 5 000*l* water through the system. [1 900kJ]

(b) A water pump has an effective suction head of 4 m and a delivery head of 60 m. Both delivery pipe and suction pipe have internal diameters of 100 mm.

Calculate:

(i) the head [64 m]

(ii) the volume of water in the pipe system if the pipes remain full [0,503 m^3J]

(iii) the work done to deliver water at this head. [321,699 kJ]

8.

(a) An electric motor is used to pump water from a borehole 30 m deep at a rate of 120*l*/min. Calculate the power required to pump the water if the efficiency of the motor is 60 per cent. [1 kW]

(b) Calculate the work done to pump water at this rate for half an hour. [1,8 MJ]

Activity 5.3

1. Under what pressure does a hydraulic accumulator with a ram diameter of 300 mm operate, if the load is 100 tons? Also calculate the energy stored if the ram is lifted 1,5 m. [1414,71 kPa; 150 kJ]
2. The load on an accumulator must be restricted to 1,5 tons and it must operate at 150 kPa. Calculate the diameter of the ram. Also calculate the volume of water which the accumulator will store if it is lifted 3 m [113 mm; 30 *l*]
3.

(a) The water used to operate a hydraulic press is under a pressure of 180kPa. Calculate the force exerted by the ram if it has a diameter of 500mm. [35,343 kN]

(b) The plunger of a three-stroke pump used in the above-mentioned press, has a diameter of 75 mm and a stroke length of 300 mm. The press must perform one operating stroke of 375 m every minute. Calculate the speed at which the pump must operate if there is a slip of 6%. [19,692r/min]

(c) An accumulator is fitted between the press and the pump. Calculate the smallest permissible content of the accumulator if the operating stroke of the press takes 5 seconds. [0,0676 m^3]

Self Check

I am able to:	YES	NO
• Analytically solve practical problems which deal with the determination of:		
o The volume of liquid required per stroke by the press (slit p be taken into account).	○	○
o The diameter of the ram of the press or force exerted by the ram.	○	○
o The pressure in the liquid to overcome a given load.	○	○
o The work done by the press (efficiency to be taken into account).	○	○
o The volume of liquid delivered per stroke or in a certain time (slip to be taken into account).	○	○
o The pressure in the liquid, plunger diameter and force on the plunger.	○	○
o The rotational frequency of a pump to deliver a given volume of water.	○	○
o The power required and efficiency with input power given in the case of single, double-and three-cylinder single-acting pumps.	○	○
o The volume of liquid delivered by the accumulator per working stroke, of the machine it serves or the volume delivered in a given time.	○	○
o The diameter of the ram and the load required on the accumulator to keep the pressure required constant during the working stroke.	○	○
o The distance travelled by the accumulator ram during the working stroke.	○	○
o The transfer of pressure in the liquid between the accumulator and the machine.	○	○
o The work done and the power with efficiency and slip taken into account.	○	○
• Draw sketches which illustrate the working of an accumulator.	○	○

If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Notes

Module 6

Stress, Strain and Young's Modulus

Learning Outcomes

On the completion of this module the student must be able to:

- Do calculations on tensile and compressive stress, including the determination of cross-sectional and load or dimensions of a member to satisfy the given conditions.
- Do calculations on shear stress, including the determination of cross-sectional areas and load or dimensions of a member to satisfy the given conditions, where double shear must also be taken into account.
- State Hooke's law and define Young's modulus of elasticity.
- Do calculations and change the subject to any unknown.
- Draw stress-strain graphs, limited to the elastic limit, with clear reference to the direct proportionality between stress and strain.

6.1 Introduction

There are quite a number of practical examples where objects or parts of objects are subjected to forces. For example, machine parts, levers, structural members and so on. These forces are resisted by the internal forces of cohesion between the particles of the material. The resistance that is in the material is called stress.

6.2 Load

The external force or combination of forces acting upon a member or structure is called a load (F).

There are many examples in practice where parts are loaded, including machine parts, levers and structural members.

Such parts are in a state of rest under a system of external forces and the reactions at the supports.

As each member itself is in equilibrium, the resultant of all the forces acting upon it must be zero, but the forces tend to deform (or distort) the object or part. This action is resisted by the internal forces of cohesion between the particles of the material.

The resistance of this internal reaction (due to external force) is known as stress (**u**).

The simplest form of load is direct tensile or compressive load.

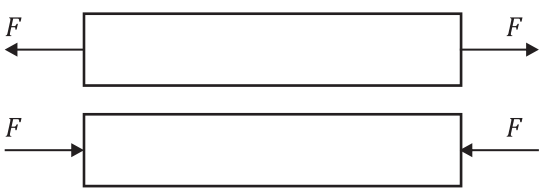


Figure 6.1

Examples of tensile load: cables, anchor cables or poles, and tie-bars. Examples of compressive load: connecting rods of engines, pillars and columns, minestruts, table and chair legs.

In structural frames (roof frames, steel constructions, and so on), some of the members of the frame structure will be subjected to tensile loads while others will be subjected to compressive loads.

6.3 Stress

At any section ** of the bar shown in **Figure 6.2**, the total force at the section must be equal to the load F.

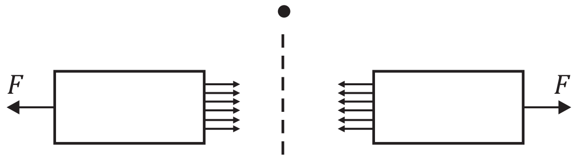


Figure 6.2

This force is distributed evenly over the internal cohesive forces known as stress (u).

If the bar is cut through any section **, each section will be in equilibrium under the action of the external force F and the stress at the section. The load F is therefore distributed evenly through the whole length of the bar.

The force transmitted over any section, perpendicular to the working line of the force, divided by the area of this section, is called the intensity of the stress or, simply, stress.

Stress can therefore be defined as the load per unit area.

$$Stress\ (\sigma) = \frac{load(F)}{area(A)}$$

From the above, it is quite clear that the unit for stress is N/m^2 or $N.m^{-2}$.

$$\therefore \sigma = \frac{F}{A}(Pa)$$

As $1\ N.m^{-2} = 1$ Pascal (Pa), the unit for stress can also be given as the Pascal.

Definition: The Pascal
The Pascal is the pressure that is produced if a force of 1 N is applied evenly and perpendicularly to an area of 1 m^2.

6.3.1 Different types of stress

- **Tensile stress**
  Tensile stress is produced when an object is subjected to a tensile load.

Figure 6.3a

- **Compressive stress**
  Compressive stress is produced when an object is subjected to a compressive load.

Figure 6.3b

- **Shear stress**
  Shear stress is produced when an object is subjected to a tensile or compressive load in such a way that this load will tend to shear the material at a certain surface. In **Figure 6.3c,** the rivet for example is subjected to a shear stress while the plate is subjected to a tensile stress.

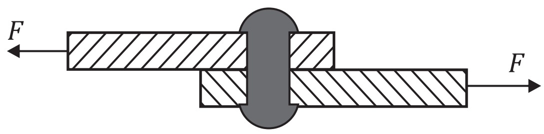


Figure 6.3c

6.4 Strain (deformation)

If the material of an object is subjected to a sufficient load, stress is produced in the material and changes the shape of the object.

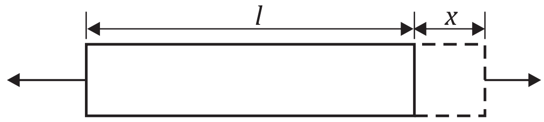


Figure 6.4a

The nature of the deformation (e) depends on the type of load acting on the object. If a tensile load is applied to a bar (**Figure 6.4a**), the load will tend to extend the bar, or deform the bar to make it longer.

Although deformation also occurs in the thickness of the bar, we will only consider deformation in the direction of the applied load. This longitudinal deformation is known as extension or change in length (x).

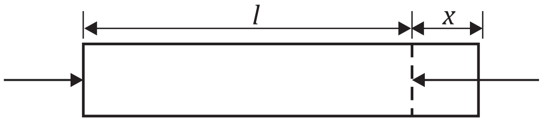


Figure 6.4b

Reduction in length takes place when a compressive load is applied to an object.

The change in length per unit length for a homogeneous material and object will be uniform, since the intensity of the stress is the same over the whole area and length. The total change in length depends on the original length.

Strain may therefore be defined as the change in length per unit length.
change in length (x)

$$Strain\ (\varepsilon) = \frac{change\ in\ length(x)}{original\ length(l)}$$

$$\therefore\ \varepsilon = \frac{x}{l}$$

It is clear that strain is a ratio of length and therefore contains no units.

When an object is subjected to shear stress (**Figure 6.4c**), the strain is represented by the symbol $\emptyset = \frac{x}{l}$ to distinguish it from strain caused by tensile stress or compressive stress.

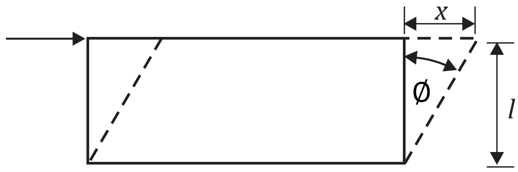


Figure 6.4c

6.5 Elasticity

Elasticity is the property of a material or object to return to its original form when the load causing the strain is removed. If the elastic limit is exceeded, permanent deformation occurs and the object will not return to its original dimensions.

6.6 Elastic limit

It is the stress up to which complete recovery from the strain takes place upon removal of the load.

6.7 Hooke's law

Hooke's law states that for an elastic object, the strain is directly proportional to the stress causing it. The point at which the strain is no longer proportional to the stress is called the limit of proportionality.

6.8 Young's elastic modulus

If stress is proportional to strain (Hooke's law), the ratio of stress divided by strain will be a constant for a given material within the limit of proportionality. This constant is known as Young's modulus, for tensile and compressive stresses, and is denoted by the symbol E.

Young's modulus $(E) = \frac{stress\ (\sigma)}{strain\ (\varepsilon)}$

$$\therefore E = \frac{\sigma}{\varepsilon}\ (Pa)$$

The unit for Young's modulus is therefore the same as for stress - the Pascal.

For shear stress this constant does not have the same value as E for the same material and it is known as the modulus of rigidity. It is denoted by the symbol G.

Modulus of rigidity $(G) = \frac{shear\ stress\ (r)}{shear\ strain\ (\emptyset)}$

$$\therefore G = \frac{r}{\emptyset}$$

6.9 Behaviour of mild steel in tension

A convenient way of demonstrating elastic behaviour is to plot a graph of the results of a simple tensile test carried out on a thin mild steel rod. The rod may be hung vertically and a series of forces which are gradually increased, applied at the lower end.

Two gauge points are marked on the rod and the distance between them measured after each force increment has been added. The test is continued until the rod breaks.

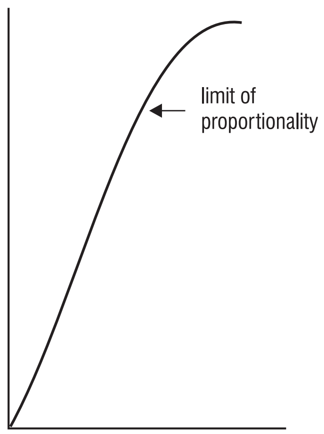


Figure 6.5

The stress and strain values at each incremental stage are calculated and plotted as shown in **Figure 6.5**. If a graph is drawn of stress against strain when the load is gradually applied, then the first portion of the graph will be a straight line. The slope of this straight line is the constant of proportionality known as the modulus of elasticity and the portion over which Hooke's law is obeyed.

Worked Example 6.1

A tensile force of 20 kN is applied to a rectangular bar 20 mm wide and 10 mm thick. The length is 2 m and Young's modulus for steel is 200 GPa.

Calculate:

(a) the stress

(b) the final length due to the tensile load.

Solution:

Given: load = 20×10^3 N; width = 0,02 m; thickness = 0,01 m; length = 2 m; E = 200×10^9 Pa

(a) Stress $= \frac{load}{area}$

$$\sigma = \frac{F}{A}$$

$$= \frac{20 \times 10^3}{0{,}02 \times 0{,}01}$$

$$= 100 \text{ MPa}$$

(b) Young's modulus $= \frac{stress}{strain}$

$$E = \frac{\sigma}{\varepsilon}$$

$$\therefore \varepsilon = \frac{\sigma}{E}$$

$$= \frac{100 \times 10^6}{200 \times 10^9}$$

$$= 0{,}000\ 5$$

Strain $= \frac{extension}{original\ length}$

$$\therefore \varepsilon = \frac{x}{l}$$

$$\therefore x = \varepsilon l$$

$$= 0{,}0005 \times 2$$

$$= 0{,}001 \text{ m}$$

Final length = original length + extension $= 2 + 0{,}001$

$$= 2{,}001 \text{ m}$$

Worked Example 6.2

During a tensile test on a round bar with a 20 mm diameter, a load of 60 kN causes an extension of 0,193 mm. If the original length was 205 mm, calculate:

(a) the stress
(b) the strain
(c) Young's modulus of elasticity

Solution:

Given: diameter = 0,02 m; load = 60 x 10^3 N; extension= 0,193 mm; original length = 205 mm

(a) Stress $= \frac{load}{area}$

$\therefore \sigma = \frac{60\times10^3}{\frac{\pi}{4}(0{,}02)^2}$

$= 191$ MPa

(b) Strain $= \frac{extension}{original\ length}$

$\therefore \varepsilon = \frac{0{,}193}{205}$

$= 0{,}000\ 941$

(c) Young's modulus $= \frac{stress}{strain}$

$E = \frac{191\times10^6}{0{,}000\ 941}$

$= 203$ GPa

Worked Example 6.3

A bolt 500 mm long has a diameter of 120 mm for a length of 250 mm and a diameter of 113,5 mm for the remaining length. If the bolt is subjected to a maximum tensile load of 1 MN, calculate the total extension of the bolt.
E = 208 GPa.

Solution:

The extension of the different sections must be calculated separately.

Stress in thicker portion:

$\sigma = \frac{F}{A}$

$\therefore \sigma = \frac{1\times10^6}{\frac{\pi}{4}(0{,}12)^2}$

$= 88{,}42$ MPa

Strain in thicker portion:

$\varepsilon = \frac{\sigma}{E}$

$= \frac{88{,}42\times10^6}{208\times10^6}$

$= 0{,}000\ 425$

Extension of thicker portion:

$\varepsilon = \frac{x}{l}$

$\therefore x = \varepsilon\, l$

$= 0{,}000\ 425 \times 250$

$= 0{,}106$ mm

Stress in thinner portion:

$\sigma = \frac{F}{A}$

$\therefore \sigma = \frac{1\times10^6}{\frac{\pi}{4}(0{,}113{,}5)^2}$

$= 98{,}84$ MPa

Strain in thicker portion:

$\varepsilon = \frac{\sigma}{E}$

$= \frac{98{,}84\times10^6}{208\times10^9}$

$= 0{,}000\ 475$

Extension of thicker portion:

$\varepsilon = \frac{x}{l}$

$\therefore x = \varepsilon\, l$

$= 0{,}000\ 425 \times 250$

$= 0{,}119$ mm

Total extension $= 0{,}106 + 0{,}119$

$= 0{,}225$ mm

Worked Example 6.4

The round bar in **Figure 6.6** is subjected to a tensile force of 120 kN. Calculate what the diameter of the middle portion would be if the stress there is not to exceed 215MN.m^{-2}.

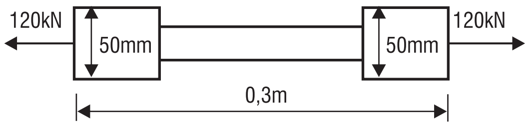


Figure 6.6

Solution:

Stress in middle portion $= \frac{load}{area}$

$$\therefore 215 \times 10^6 = \frac{120 \times 10^3}{\frac{\pi}{4} D^2}$$

$$\therefore D^2 = 7{,}106 \text{ x } 10^{-4}$$

$$D = 0{,}026\ 66\text{mm}$$

$$= 26{,}66 \text{ mm}$$

Worked Example 6.5

The following data was recorded during a tensile test on a circular cross-section of a mild steel test piece. The initial diameter of the cross-section is 25mm and the gauge length is 250 mm.

Load kN	Extension mm
20	0,05
40	0,10
60	0,16
80	0,21
100	0,31
120	0,36
140	0,38
150	0,41
170	0,44
172	0,47
174	0,50

Table 6.1

Plot the stress/strain graph and from it calculate:

(a) the modulus of elasticity of the material
(b) the limit of proportionality.

Solution:

The stresses (σ) are obtained by dividing the loads (W) by the original cross-sectional area (A) of the test piece, and the strains (ε) are obtained by dividing the extensions (x) by the original length (l).

Area of cross-section (A) $= \frac{\pi}{4} \text{ x } (0,025)^2$

$= 4{,}9 \text{ x } 10^{-3} \text{ m}^2$

We now obtain the following co-ordinates:

Stress MPa	Strain	Stress MPa	Strain
40,74	0,0002	346,32	0,0018
81,49	0,0004	350,40	0,002
122,23	0,0006	354,47	
162,97	0,0008		
203,72	0,0010		
244,46	0,0012		
285,21	0,0015		
305,58	0,0016		
325,95	0,0017		

Table 6.2

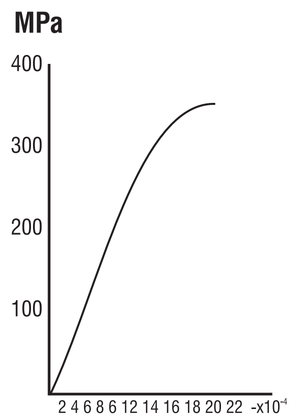


Figure 6.7

Continued overleaf ...

(a) The modulus of elasticity(E) is determined from the slope of the straight portion of the graph. A change in stress of 244,46MPa produces a strain of 0,0012.

∴ Modulus of elasticity $E = \frac{244{,}46 \times 10^6}{0{,}0012}$

$= 203{,}7 \text{ GPa}$

(b) According to the graph, the stress at the limit of proportionality, that is, the end of the straight line portion, is 264MPa.

6.10 Summary of formulae

- Stress $= \frac{load}{area}$

  $\therefore \sigma = \frac{F}{A}$

- Strain $= \frac{change\ in\ length}{original\ length}$

  $\therefore \varepsilon = \frac{x}{l}$

for tensile and compressive stress and $x = \frac{x}{l}$ for shear stress

- Young's modulus $= \frac{stress}{strain}$

  $\therefore E = \frac{\sigma}{\varepsilon}$

for tensile and compressive stress

- The modulus of rigidity $= \frac{shear\ stress}{shear\ strain}$

  $\therefore G = \frac{T}{\emptyset}$

Activity 6.1

1. A hollow cast-iron column has an external diameter of 300 mm and an internal diameter of 250 mm, and is 3,06 m long. By what amount will it become shorter under a load of 600 kN? Take *E* for cast-iron as 120 x 103MPa. [0,708 4 mm]
2. A steel wire 3,08 m long has a cross sectional area of 8 080 mm^2. It is hung vertically and stretches 0,391 mm when loaded with 2,5 kN.

Calculate:

   (a) the stress [309,406 kPa.]
   (b) the strain [$1{,}269 \times 10^{-4}$]
   (c) the modulus of elasticity. [2,438 GPa.]

3. A round steel bar 4,2 m in length is subjected to a tensile load of 100 kN. The bar has a diameter of 39,1 mm over a length of 0,923 m, a diameter of 32,25 mm over a length of 1,32 m, and a diameter of 25,3 mm over the remaining length. Calculate the total extension of *E* = 190 GPa.[3,304 mm]
4. The steel spokes of a bicycle wheel have a cross-sectional area of 2,238mm^2 and a length of 308 mm. If the nut of the spoke is tightened until the spoke stretches 0,25mm, determine the tensile force on each spoke. Take *E* for the spoke material to be 248 GPa. [450,506 N]
5. A tie-bar has to carry a load of 150 kN. What must the width of the bar be if there is a rivet hole of diameter 22,21 mm on its centre line? The thickness of the bar is 12,8mm and the working stress is 78,3 MPa. [171,87 mm]
6. A square steel bar with sides 10 mm long is subjected to a compressive load of 100kN.

Calculate:

   (a) the stress in the material [4,444 MPa]
   (b) the strain, if Young's modulus for steel is 200 GPa. [0,000 02]

Continued overleaf ...

7. A tensile force of 70 kN is applied to a round aluminium bar, causing a stress of 24,757 MPa. The original length is 5 m and Young's modulus for aluminium is 70 GPa.

Calculate:

(a) the diameter of the bar [60mm]
(b) the strain [0,000 354]
(c) the increase in length due to the load. [1,768 36 mm]

8. A brass bar with a 10 mm diameter is rigidly secured at the top while hanging vertically. A mass of M kg is clamped to the bottom so that the effective length of the bar is 2 m. Calculate the magnitude of the mass if it causes an extension of 0,124mm. Take *E* for brass as 70 GPa. [34,086 kg]
9. A round brass bar with a 20 mm diameter and 0,75 m length is machined to a square of 10 mm side length over a length of 0,25 m. If the stress in the square section is not to exceed 300 MPa, calculate:
(a) the stress in the round section [95,493 MPa]
(b) the total extension of the bar if Young's elastic modulus for brass is 70GPa. [1,754mm]
10. The ratio of external diameter to internal diameter of a pipe is 1,25:1. The length of the pipe is 3,05 m and it is loaded axially by a compressive load of 600 kN. Calculate the external and internal diameters of the pipe if it is shortened by 0,835 1 mm under this load. Take the elastic modulus as 124GPa. [200 mm; 250 mm]
11. Two flat steel strips 35 mm wide and 6 mm thick are joined by a single rivet with a 12mm diameter, as shown in **Figure 6.8**. A load of 20 kN is applied as shown.

Calculate:

(a) the tensile stress in the solid part of the strip [95 ,238 MPa]
(b) the shear stress in the rivet [176,539 MPa]
(c) the tensile stress in the section opposite the rivet. [144,9 MPa]

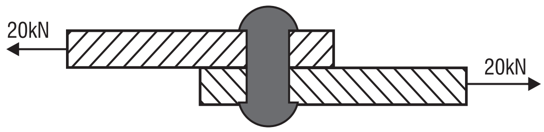


Figure 6.8

12. The following data is given for a cold worked carbon steel tested in a tensile test:
Original diameter: 12,725 mm.
Original length: 50,8 mm.
Plot a load-extension graph. From your graph determine the:
(a) proportional limit
(b) modulus of elasticity.

Load (kN)	Extension (mm)	Load (kN)	Extension (mm)
5,516	0,0102	40,479	0,762
11,743	0,0203	53,14	0,1016
16,992	0,0305	60,5	0,127
22,108	0,0406		
27,401	0,0508		
40,479	0,0762		

Table 6.3

13. The following table gives observations from the tensile test of the destruction of a round piece of mild steel 20 mm in diameter and 200 mm between gauge points.
Plot the stress-strain graph and determine from the graph:
(a) the limit of proportionality
(b) the modulus of elasticity.

Load (kN)	Extension (mm)
50	0,119
100	0,243
150	0,368
160	0,493
170	0,616
180	0,740
190	0,869
200	1,130

Table 6.4

14. The following results were obtained in a tensile test on a mild steel specimen 20 mm wide by 10 mm thick, the gauge length being 200 mm.

Load (kN)	Elongation (mm)
16	0,066
32	0,133
48	0,198
64	0,264
68	0,281
72	0,304

Table 6.5

Continued overleaf ...

Plot the load-extension diagram using the following scale:
10 divisions on x-axis = 5 mm extension and
10 divisions on y-axis = 10 kN load.
On the same graph paper and using the following scale:
10 divisions on x-axis = 0,5 mm extension and
10 divisions on y-axis = 10 kN load, re-plot the elastic proportion of the graph.
Using your graphs, determine the following:
(a) Young's modulus of elasticity
(b) the elastic limit.

Self Check

I am able to:	YES	NO
• Do calculations on tensile and compressive stress, including the determination of cross-sectional and load or dimensions of a member to satisfy the given conditions.	○	○
• Do calculations on shear stress, including the determination of cross-sectional areas and load or dimensions of a member to satisfy the given conditions, where double shear must also be taken into account.	○	○
• State Hooke's law and define Young's modulus of elasticity.	○	○
• Do calculations and change the subject to any unknown.	○	○
• Draw stress-strain graphs, limited to the elastic limit, with clear reference to the direct proportionality between stress and strain.	○	○

If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Notes

Module 7 Heat

Learning Outcomes

On the completion of this module the student must be able to:

- Change from Celsius to the kelvin temperature scale and vice versa.
- Analytically solve practical problems on the expansion of solids due to temperature changes (area and volume expansion).
- Analytically solve practical problems on volumetric expansion of liquids as a result of a rise in temperature.
- Be aware of the anomaly in the expansion of water.
- Reproduce the laws of Boyle and Charles in the form of specified mathematical equations.
- Do calculations which entail applications of the laws of Boyle and Charles and the combination of the two laws.

7.1 Introduction

In physics, heating is transfer of energy, from a hotter body to a colder one, other than by work or transfer of matter. It occurs spontaneously whenever a suitable physical pathway exists between the bodies. The pathway can be direct, as in conduction and radiation, or indirect, as in convective circulation.

Heat is a form of energy. The unit of energy and therefore of heat is the joule. Heat may be used to do work.

7.2 Temperature scales

Temperature indicates the degree of heat and not the quantity of heat that a substance contains.

To indicate the degree of heat, different temperature scales are used, two of which are found in the SI system and used in this course.

7.2.1 The Celsius scale

The Celsius scale is used in ordinary temperature readings. It is divided into one hundred divisions between the freezing point of pure water (0 °C at sea level) and the boiling point of pure water (100 °C at sea level).

7.2.2 The kelvin scale

The kelvin scale is the unit scale for thermodynamic temperature.

On this scale zero kelvin (K) is the temperature at which an ideal gas would theoretically have no volume. That is, however, not possible, as gases become liquid before this temperature can be reached.

On the kelvin scale, 0 kelvin corresponds to -273 °C and 273 K is equal to 0 °C.

At -273 °C, no substance would have any heat energy left.

7.2.3 Relation between degrees Celsius and kelvin

Let t = temperature in °C; T = thermodynamic temperature in K and T_o = 273.

The relation between thermodynamic temperature and Celsius temperature is then given by: $t = T - T_o$

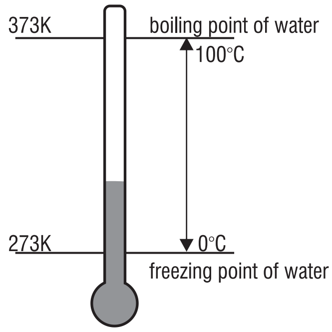


Figure 7.1

Worked Example 7.1

(a) Convert 30 °C to degrees kelvin.
(b) convert 293 K to degrees Celsius.

Solution:

(a) t = T- To
$30 = T - 273$
$T = 273 + 30$
$T = 303\ K$

(b) t = T- To
$t = 293 - 273$
$t = 20\ °C$

7.3 Expansion of solids

When a solid is heated there is an increase in its dimensions; its length increases, its surface area increases and its volume increases. The inverse is also true; if heat is taken away, there will be a decrease in the above dimensions.

7.3.1 Coefficient of linear expansion

The coefficient of linear expansion for solids is the change in length per unit length per degree change in temperature.

When a solid is subjected to temperature changes the change in length (δl) will be proportional to the original length and the change in temperature (δt) $\therefore \delta 1 \propto 18t.$

Let l_1 = length of bar at temperature t_1 °C
l_2 = length of the bar at temperature t_2 °C
α = coefficient of linear expansion of the substance

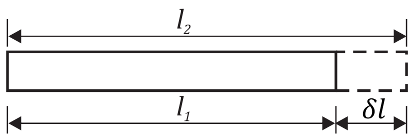


Figure 7.2

then $l_2 - l_1$ = change in length $\delta 1$
$t_2 - t_1$ = change in temperature δt and
$\delta 1 = \alpha l_1 \times \delta t$

or $\alpha = \frac{\delta l}{l_1 \times \delta t} = \frac{l_2 - l_1}{l_1(t_2 = t_1)}$

also $l_2 = l_1 + \delta l$

Worked Example 7.2

Two rods, one made of steel and the other of aluminium, are exactly 5 m long at a temperature of 573 K. Calculate the difference between the lengths at a temperature of 273 K α of steel = 12 x 10^{-6}/K and α of aluminium = 24 X 10^{-6}/K.

Solution:

Given:

	Steel	Aluminium
	l = 5 m	l = 5 m
	T_1 = 273	T_1 = 273 K
	T_2 = 573	T_2 = 573 K
	α_s = 12 x 10^{-6}/K	α_a = 24 x 10^{-6}/K

Change in length for steel

$$= \alpha_s l(T_2 - T_1)$$
$$\delta l_s = 12 \times 10^{-6} \times 5(573 - 273)$$
$$= 0{,}018 \text{ m}$$

Change in length for aluminium

$$= \alpha_a l(T_2 - T_1)$$
$$\delta l_a = 24 \times 10^{-6} \times 5(573 - 273)$$
$$= 0{,}036 \text{ m}$$

Difference between lengths $= \delta l_a - \delta l_s$

$= 0{,}036 - 0{,}018$

$= 0{,}018$ m

$= 18$ mm

Worked Example 7.3

A bar is 1,022 6 m long at a temperature of 5 °C and 1,023 7 m at 94 °C. Calculate the value of the coefficient of linear expansion.

Solution:

Given: l_1 = 1,022 6 m t_1 = 5 °C

l_2 = 1,023 7 m t_2 = 94 °C

$$\delta l = l_2 - l_1$$
$$= 1{,}023\,7 - 1{,}022\,6$$
$$= 1{,}1 \times 10^{-3} \text{ m}$$
$$\delta t = t_2 - t_1$$
$$= 94 - 5$$
$$= 89 °\text{C}$$
$$\alpha = \frac{\delta l}{l_1 \delta t}$$
$$= \frac{1{,}1 \times 10^{-3}}{1{,}022\,6 \times 89}$$
$$= 12{,}086\,4 \times 10^{-6} \text{ /°C}$$

7.3.2 Coefficient of area expansion

The coefficient of area (or superficial) expansion (β) of a solid is the change in area per unit area per degree change in temperature.

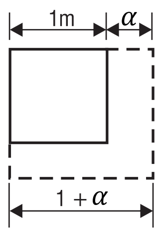


Figure 7.3

Let A be a square sheet of metal with side length 1m.

Original area = 1 m^2

Let α be the coefficient of linear expansion per degree rise in temperature. Then the new length of the sheet metal will be

$= (1 + \alpha)$ m and the new area $= (1 + \alpha)^2$ m^2

$= (1 + 2\alpha + \alpha^2)$ m^2

Continued overleaf ...

α is small enough to be ignored, so the new area $= (1+2\alpha)\ \text{m}^2$

∴ Increase in area $= 2\ \alpha \text{m}^2$

If the temperature of a unit area therefore increases by $1\ °\text{C}$, the area increases by $2\alpha\ \text{m}^2$.

Coefficient of area expansion (β) = 2 x coefficient of linear expansion

∴ $\beta = 2\alpha$ (approximately)

When a solid is subjected to a change in temperature, the change in area (δA) is proportional to the original area and the change in temperature (δt).

∴ $\delta A \propto A\delta t$

Let A_1 = area at temperature t_1 °C

A_2 = area at temperature t_2 °C

β = coefficient of area expansion

then $A_2 - A_1$ = change in area δA

$t_2 - t_1$ = change in temperature δt

$\delta A = \beta A\ \delta t$

or $\beta = \frac{\delta A}{A_1(t_2 - t_1)}$

$= \frac{A_2 - A_1}{A_1(t_2 - t_1)}$

also $A_2 = A_1 + \delta A$

Worked Example 7.4

A brass sheet 400 mm square is at 10 °C and has a linear coefficient of expansion of 20 x 10^{-6} °C. Calculate the increase in area if the temperature increases to 100 °C.

Solution:

Given: $A1 = 400^2\ \text{mm}^2$; $\delta t = (100 - 10) = 90\ °\text{C}$; $\alpha = 20 \times 10^{-6}/°\text{C}$

$\beta = 2\alpha$

$= 2 \times 20 \times 10^{-6}$

$= 40 \times 10^{-6}\ /°\text{C}$

$\delta A = \beta A_1\ \delta t$

$= 40 \times 10^{-6} \text{ X } 400^2 \times 90$

$= 576\ \text{mm}^2$

Worked Example 7.5

The area of a brass sheet is 1 cm^2 at 0 °C and 1,003 6 cm^2 at 100 °C. Calculate the coefficient of linear expansion of brass.

Solution:

Given: $A1 = 1\ \text{cm}^2$; $A_2 = 1{,}003\ 6\ \text{cm}^2$; $t_1 = 0\ °\text{C}$; $t_2 = 100\ °\text{C}$

$\beta = \frac{A_2 - A_1}{A_1(t_2 - t_1)}$

$= \frac{1{,}003\ 6 - 1}{1(100 - 0)}$

$= 3{,}6 \times 10^{-5}$

Also $\beta = 2\alpha$

∴ $\alpha = 18 \times 10^{-6}\ /°\text{C}$

7.3.3 Coefficient of cubic expansion

The cubic expansion coefficient of a solid is the increase of a unit volume of the substance per degree change in temperature.

Let A be a metal cube with side lengths of 1 m
Original volume = 1m^3
Let α be the linear expansion coefficient per degree rise in temperature.

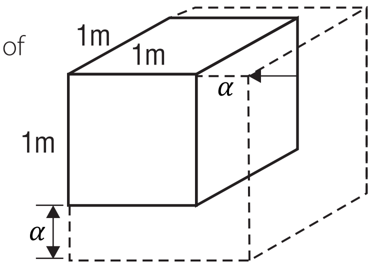


Figure 7.4

Then the new side length of the cube will be

$= (1 + \alpha)\text{ m}$

and the new volume $= (1 + \alpha)^3\text{ m}^3$

$= (1 + 3\alpha + 3\alpha^2 + \alpha^2)\text{m}^3$

α^2 and α^3 are small enough to be ignored, so

the new volume $= (1 + 3\alpha)\text{ m}^3$

Change in volume $= 3\alpha\text{m}^3$

If the temperature rises by $1\ °\text{C}$, the volume will increase by $3\alpha\text{ m}^3$.
Volume (cubic) expansion coefficient (γ) = 3 x linear expansion coefficient (α).
$\gamma = 3\alpha$ (approximately)
When a solid is subjected to a change in temperature, the change in volume (δV) is proportional to the original volume and the change in temperature (δt).

$\delta V \propto V\delta t$

Let $V_2 - V_1$ = volume at temperature $t_1\ °\text{C}$
V_2 = volume at temperature $t_2\ °\text{C}$
γ = volume (cubical) expansion coefficient
then $V_2 - V_1$ = change in volume δV
$t_2 - t_1$ = change in temperature δt

$\delta V = \gamma V_1 \delta t$

or $\gamma = \frac{\delta V}{V_1 \delta t}$

$= \frac{V_2 - V_1}{V_1(t_2 - t_1)}$

also $V_2 = V_1 + \delta V$

Worked Example 7.6

A spherical container has a volume of 1m³. Calculate the increase in volume if the temperature increases by 80 °C and the linear expansion coefficient of the material is 17×10^{-6} °C.

Solution:

Given: $V_1 = 1\text{m}^3$; $\delta t = 80\ °\text{C}$; $\alpha = 17 \times 10^{-6}/°\text{C}$

$\gamma = 3\alpha$
$= 3 \times 17 \times 10^{-6}$
$= 51 \times 10^{-6}\ /°\text{C}$

$\delta V = \gamma V \delta t$
$= 51 \times 10^{-6} \times 1 \times 80$
$= 4{,}08 \times 10^{-3}\text{ m}^3$

Worked Example 7.7

A spherical container has a volume of 1 m^3. The temperature is increased by 60 °C. The linear expansion coefficient is 17 x 10^{-6} /°C.
Calculate:
(a) the increase in volume
(b) the final volume
(c) the final diameter

Solution:

Given: V_1 = 1 m^3; δt = 60 °C; α = 17 x 10^{-6}/°C

(a)
$$\delta V = \gamma V_1 \delta t$$
$$= 3\alpha V_1 \delta t$$
$$= 3 \times 17 \times 10^{-6} \times 1 \times 60$$
$$= 3{,}06 \times 10^{-3}\ m^3$$

(b)
$$V_2 = V1 + \delta V$$
$$= 1 + 3{,}06 \times 10^{-3}$$
$$= 1{,}003\ 06\ m^3$$

(c)
$$V = \pi/6D^3$$
$$1{,}003\ 06 = \pi/6D^3$$
$$D = 1{,}242\ m$$

7.3 Important formulae for solids and liquids

- $T = t - T_o$
- $\delta l = \alpha l \delta t$
- $\delta A = \beta A \delta t$
- $\beta = 2\alpha$
- $\delta V = \gamma V \delta t$
- $\gamma = 3\alpha$

7.4 Expansion of liquids

As liquids and gasses take the shape of the container in which they are held, only their change in volume can be examined. It is not so simple to measure the expansion of a liquid, as the container itself will also expand.

7.4.1 Absolute and apparent expansion

The expansion of a liquid may be illustrated by means of a glass flask fitted with a rubber bung through which there is a length of glass tubing. The flask is filled with water or another fluid and the rubber bung is pressed in until the fluid level rises in the glass tubing.

On plunging the flask into hot water, the water level in the tube will drop slightly and then start to rise steadily (**Figure 7.5**).

The initial drop in level results from the thermal expansion of the glass flask, which becomes hotter before the heat is conducted through the glass into the liquid.

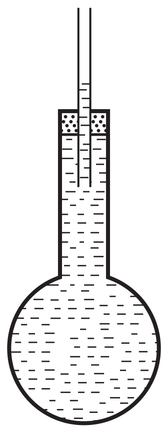


Figure 7.5

- The absolute expansion coefficient of a liquid is the amount by which the original volume of the liquid increases per degree rise in temperature.
- The apparent expansion coefficient of a liquid is the amount by which the liquid expands per degree rise in temperature irrespective of whether the container itself had expanded or not. Absolute expansion = apparent expansion + expansion of container.

Continued overleaf ...

Worked Example 7.8

A glass flask is filled with 40 ems mercury at a temperature of 18 °C. Calculate the volume that overflows if the temperature is increased to 38 °C. The linear expansion coefficient of glass (α_g) is 9 x 10^{-6} /K, and the volumetric expansion coefficient of mercury (ךm) is 18 x 10^{-5} /K.

Solution:

Given: Volume mercury = 40 mm^3 ; t_1 = 18 °C; t_2 = 38 °C; α_g = 9 x 10^{-6} /K; ךm = 18 x 10^{-6} /K.

Absolute volumetric expansion of mercury: δV_m = ך$mV_1\delta t$
= 18 x 10^{-5} x 40 x 20
= 0,144 cm^3

Volumetric expansion of glass flask: δV_g $3\alpha g V_1 \delta t$ =
= 3 x 9 x 10^{-6} x 40 x 20
= 0,021 cm^3

Volume overflowing (apparent volume) = absolute volumetric expansion of mercury - volumetric expansion of flask δV_m = δV_g
= 0,144 - 0,021 6
= 0,122 4 cm^3

7.4.2 The anomalous expansion of water

Not all substances expand when heated. Water is an example, as it shrinks during a certain temperature range if it is heated. Take a glass flask and fill it with water, and fit a rubber bung with a glass tube and a thermometer.

Press the bung into the opening of the flask until the level of the water is a short distance up the tube (**Figure 7.6**).

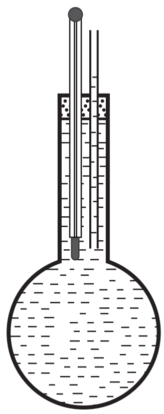


Figure 7.6

Put the flask in melting ice at 0 °C. Note the level of the water after about 10 minutes or as soon as the meniscus no longer changes its position. The flask with its content will now be at 0 °C.

Slowly heat the container of melting ice and carefully note the position of the meniscus. You will see that the meniscus falls until a temperature of 4 °C is reached. The water has contracted, while the temperature has been increased from 0 °C to 4 °C.

Further heating of the water causes the meniscus to rise again. The rate of this rise is small initially, but increases as the temperature rises. The water therefore expands from 4 °C, but this expansion is anomalous.

The above relation between the volume of the water and the temperature is graphically represented in **Figure 7.7**.

At 4 °C, the water will have its minimum volume (the density will be at its maximum) and at 0 °C, ice is formed and there is a sudden increase in volume, which then decreases as the temperature decreases.

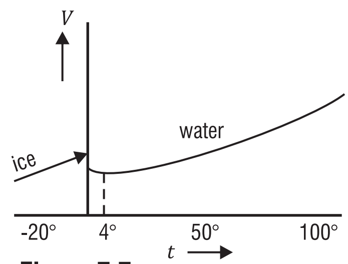


Figure 7.7

7.5 Expansion of gases

As already mentioned, each solid or liquid has its own particular coefficient of expansion. The coefficient of expansion of gases differs from that of solids and liquids in the following respects:

- All gases have the same coefficient of expansion: $\frac{1}{273}$ of the original volume at 0 °C per degree Celsius.
- The expansion of a gas is relatively large in comparison with that of solids and liquids.

Gases, because of their great compressibility and thermal expansiveness, occupy volumes that depend very sensitively on pressure and temperature. All gases obey three simple laws that relate volume to pressure and temperature.

7.5.1 Boyle's law

Boyle's law indicates the relation between volume and pressure at constant temperature and reads as follows:

- The volume of a given mass of gas is inversely proportional to the pressure to which it is subjected, if the temperature remains constant.

$$Volume\ (V) \propto \frac{1}{pressure\ (P)} \quad \text{or} \quad V = constant \times \frac{1}{P}$$

$$PV = constant$$

Boyle's law is represented graphically in **Figure 7.8**.

As any two readings of pressure and corresponding volume give the same constant, this relation may be expanded further:

$$P_1V_1 = constant \text{ and}$$
$$P_2V_2 = constant$$
$$P_1V_1 = P_2V_2 = P_3V_3$$

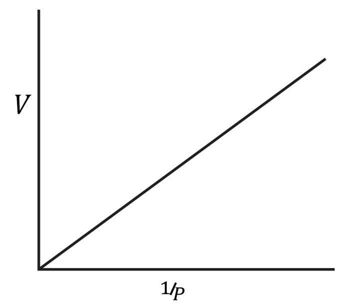


Figure 7.8

Where

V_1 = *volume of gas in* m^3 *at pressure* P_1 *in Pascal*
V_2 = *volume of gas in* m^3 *at pressure* P_2 *in Pascal*
V_3 = *volume of gas in* m^3 *at pressure* P_3 *in Pascal*

7.5.2 Charles's law

This law indicates the relation between volume and temperature at constant pressure and reads as follows:

- The volume of a given mass of gas changes by a fraction, $\frac{1}{273}$ of its volume at 0 °C for 273 each Celsius degree rise in temperature if the pressure remains constant. or:
- The volume of a given mass of gas at constant pressure is directly proportional to its thermodynamic temperature (T), where T is the thermodynamic temperature in kelvin: ($T = t + T_o$).

Charles's law is shown graphically in **Figure 5.9**.

The coefficient of cubic expansion of a gas at constant pressure is given by: $\gamma_o = \frac{V_1 - V_o}{V_o t_1}$

$$= \frac{1}{273}$$

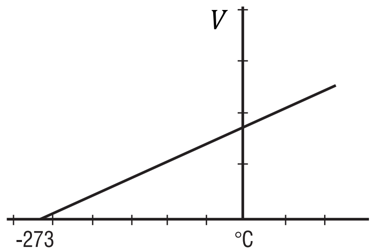


Figure 7.9

V_o = *volume of gas at 0 °C*
V_1 = *volume of gas at* t_1 *°C*
γ_o = *cubic expansion coefficient per degree Celsius*
t_1 = *temperature change from 0 °C*

Continued overleaf ...

From the above formula it follows: $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ = constant

Where $T_1 = t_1 + 273$

$T_2 = t_2 + 273$

7.5.3 The pressure law (Gay-Lussac's law)

This law gives the relation between pressure and temperature at constant volume and reads as follows:

- The pressure of a given mass of gas changes by a constant fraction, 2~3 of its pressure at 0 °C, for each degree change in temperature if the volume is kept constant.
  The coefficient of pressure increase at constant volume is given by ך$p = \frac{P_1 - P_o}{P_o t_1}$
  $= \frac{1}{273}$
  Where:
  P_o = *pressure of gas at 0 °C*
  P_1 = *pressure of gas at* t_1*°C*
  ךp = *coefficient of pressure change per degree Celsius*
  t_1 = *temperature change from 0 °C*

From the above formula it follows: $\frac{P_1}{P_2} = \frac{T_1}{T_2} = constant$

Where $T_1 = t_1 + 273$

$T_2 = t_2 + 273$

7.5.4 The combined gas laws

These gas laws give the relation between volume, pressure and thermodynamic temperature.

Consider a gas with volume V1 at a thermodynamic temperature T_1 and pressure P_1. If the temperature is kept constant while the pressure increases to P_2, the volume will change to V.

According to Boyle's law: $P_1V_1 = P_2V$

$V = \frac{P_1V_1}{P_2}$ ………………………………….. (1)

Now consider the same gas with volume V at the same pressure P_2 but heated to a thermodynamic temperature T_2. The gas will expand to a volume V_2.

According to Charles's law: $\frac{V}{V_2} = \frac{T_1}{T_2}$

$V = \frac{T_1V_2}{T_2}$ ……………………………………. (2)

From (1) and (2) it follows that: $\frac{P_1V_1}{P_2} = \frac{T_1V_2}{T_2}$ or

$\frac{P_1V_1}{T_1} = \frac{P_sV_2}{T_2}$ = constant

These equations are known as the general gas equations and are also written as $\frac{PV}{T}$ constant.

7.6 Amount of substance and molar mass

One mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0,012 kg (exactly) of ^{12}C (carbon -12).

A mole of substance is therefore a very large number of particles of that substance. This number is known as the Avagadro constant, which is represented by the symbol N_A.

The molar mass of any substance is the mass per mole of that substance (the mass of N_A particles of that substance) and is represented by the symbol M. The molar mass of ^{12}C is 12g/mole.

7.7 The general and characteristic gas equations

The general gas equation

From **Point 7.5.4** it follows that P;=constant.

The symbol used for this constant is R. R is known as the general or universal gas constant and is the same for all gases provided one mole is used.

For a mole it follows that: $PV = \pi RT$

Where $R = 8{,}31 \text{ JK}^{-1} \text{ mole}^{-1}$

The characteristics gas equation

The density and molar mass differ from gas to gas, so that th13 specific gas constant differs too.

For a specific gas, the general gas equation may also be written as: $PV = \pi RT$

Where P = *pressure in Pa*
V = *volume in* m^3
m = *mass of gas in kg*
R = *specific gas constant in J/kg.K*
T = *thermodynamic temperature in K.*

The specific gas constant for air is 287 J/kg.K.

7.8 Standard temperature and pressure (STP)

STP is a set of standard conditions for reference purposes and is a pressure of 101,3 Pa and a temperature of 273 K (0 °C).

Worked Example 7.9

Dry gas with a volume of 0,4 m 3 at a temperature of 10 °C is heated to a temperature of 125 °C while the pressure remains constant. Calculate the new volume.

Solution:

Given: $V_1 = 0{,}4\ m^3$; $t_1 = 10°C$; $t_2 = 125°C$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{0{,}4}{(10+273)} = \frac{V_2}{(125+273)}$$

$$V_2 = 0{,}53\ m^3$$

Worked Example 7.10

A certain mass of gas has a volume of 50 m^3 at an absolute pressure of 100kPa and a temperature of 210 °C.

Calculate:

(a) the volume of the gas when the temperature is lowered to -50°C while the pressure remains constant

(b) the thermodynamic temperature of the gas when the volume is reduced to 20m^3 and the absolute pressure is increased to 210 kPa.

Solution:

Given: (a) $V_1 = 50\ m^3$; $P_1 = 100\ kPa$; $t_1 = 210°C$; $t_2 = 50°C$

(b) $V_2 = 50\ m^3$; $V_2 = 20\ m^3$;
$P_1 = 100\ kPa$; $P_2 = 210\ kPa$
$t_1 = 210°C$

(a)
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{50}{210+273} = \frac{V_2}{(-50+273)}$$

$$V_2 = 23{,}085\ m^3$$

(b)
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{100\times50}{(210+273)} = \frac{210\times20}{T_2}$$

$$T_2 = 405{,}72\ m^3$$

Worked Example 7.11

Calculate the mass of gas, having a gas constant of 287 J /kg.K, that can be compressed in a cylinder so that the temperature is 100 °C, the pressure 550kPa, and the volume 0,6 m^3.

Solution:

Given: *R = 287 J/kg.K; t = 100 °C; P = 550 kPa·, V = 0,6 m^3; m = ?*

$$PV = mRT$$

$$550 \times 0,6 = m \times 287 \times (100 + 273)$$

$$m = 3 \text{ kg}$$

7.9 Important formulae for gases

- Boyle's law (constant temperature): $P_1V_1 = P_2V_2$
- Charles's law (constant pressure): $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
- Pressure law (constant volume): $\frac{P_1}{P_2} = \frac{T_1}{T_2}$
- Combined law $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
- General gas equation: $PV = nRT$
- Characteristic gas equation: $PV = mRT$
- Volume expansion coefficient: $\gamma_o = \frac{V_1 V_o}{V_o t}$
- Coefficient of pressure expansion: $\gamma_p = \frac{P_1 - P_o}{P_o t}$

7.10 Calorimetric and specific heat capacity

Calorimetry is the measurement of the amount of heat and heat transferred. One of the instruments used to measure heat transfer is the calorimeter (**Figure 7.10**). It consists of a small shiny copper or aluminium beaker in which substances are mixed. The beaker is placed in cotton wool in large beaker to minimise loss of heat.

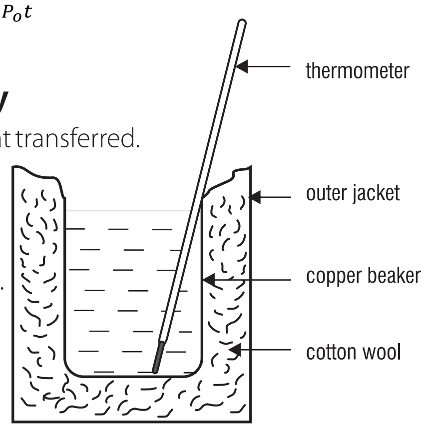


Figure 7.10

7.10.1 Heat capacity

The heat capacity of a substance is the amount of heat required to raise the temperature of the substance by 1 °C or 1 K. It may also be defined as the amount of work done on the substance per degree temperature change. The unit of heat capacity is the joule per kelvin (J/K) and the symbol for heat capacity is C.

7.10.2 Specific heat capacity

Specific heat capacity is the heat capacity per unit mass. It may also be defined as the amount of heat required to raise the temperature of 1 kg (unit mass) of the substance by 1 °C or 1 K (unit temperature). The unit for specific heat capacity is the joule per kilogram kelvin (J /kg.K) and the symbol that is used is C.

> **Note:**
> The word *specific* means *per unit mass*. The specific heat capacity of pure water is 4,187 kJ/kg. °C.

Heat capacity = mass x specific heat capacity

$$\therefore C = m.c$$

7.10.3 Amount of heat energy

The amount of heat (Q) absorbed or emitted by a substance depends on the following factors:

- The mass of the substance. A greater mass of a particular substance absorbs more heat than a smaller mass for the same change in temperature.
- The specific heat capacity of the substance. Different materials need different amounts of heat to raise their temperature by one degree Celsius or kelvin.
- The change in temperature. More heat is required to give a greater change in temperature.

Heat energy = mass x specific heat capacity x temperature change.
Heat absorbed or emitted = mass x specific heat capacity x temperature change,
or $Q = m \times c \times \Delta t$

where Q = heat energy in joule
m = mass of substance in kg
c = specific heat capacity in kJ /kg.K or kJ /kg. °C
$\Delta t = t_2 - t_1$
= temperature change in K or °C

The amount of heat energy that a body contains per kelvin (°C) is called the heat capacity of the body. A body containing 1 000 J heat energy at a temperature of 100 K therefore has a heat capacity of 10 J/K.

Worked Example 7.12

A piece of steel of mass 50 kg is cooled down so that its temperature decreases from 973 K to 298 K. If the specific heat capacity of steel is 0,377kJ/kg.K, calculate the amount of heat energy released by the steel.

Solution:

Given: *m = 50 kg; T2 = 973; T1 = 298;*
c = 0,377 kJ/kg.K; Q = ?
Q = m x c x Δt
= 50 x 0,377 x (973 - 298)
= 12,724 MJ

7.10.4 The law of conservation of energy

Energy cannot be created or destroyed; it can, however, be converted from one form into another.

Energy may also be transferred from one substance to another. The heat energy emitted by one substance is equal to the heat energy absorbed by the other substance. Heat flows from a hot body to a cold body until both bodies have the same temperature.

When solving problems where different substances at different temperatures are added together, the law of conservation of energy must be applied.

Worked Example 7.13

Iron bars with a total mass of 3·,5 kg are heated to 500 K and placed immediately in 28 kg water at 285 K. Calculate the final temperature of the iron bars and water. The heat capacity of steel is 461 kJ /kg.K.

Solution:

Given: *Mass of iron bars = 3,5 kg*
temperature of iron bars = 500 K
mass of water = 28 kg
initial temperature of water = 285 K
c = 461 kJ /kg.K
Heat released by iron bars = heat gained by water.
$m_o \times m_o \times \Delta t = m_\omega \times m_\omega \times \Delta t$
3,5 x 461 x (500 - T) = 28 x 4,187x (T - 285)
806 750 - 1613,5 T =117,236 T - 33 412,26
1 730,736 T = 840 162,26
T = 485,436 K

Worked Example 7.14

Oil with a density of 0,8 kg/l and a specific heat capacity of 3,14 kJ /kg. °C is used to temper 4 kg steel at a temperature of 667 °C. The temperature of the steel decreases to 27 °C. The rise in temperature of the oil may not exceed 25 °C.

Calculate:

(a) the heat released by the steel if the specific heat capacity of steel is 460 J/kg. °C

(b) the amount of oil required if it absorbs all the heat released by the steel.

Given:	**Steel**	**Oil**
	m = 4 kg	m = ?
	T_2 = 667 °C	Δt = 25 °C
	t_1 = 27 °C	c = 31,14 kJ/kg. °C
	c = 0,460 kJ/kg. °C	

(a) Heat released by steel

$= m \times c \times (t_2 - t_1)$

$= 4 \times 460 \times (667 - 27)$

$= 1177{,}6\ kJ$

(b) $1177{,}6 = m \times c \times (t_2 - t_1)$

$= m \times 3{,}14 \times 25$

$m = 15\ kg$

but density $= \frac{mass}{volume}$

volume $= \frac{15}{0{,}8}$

$= 18{,}75l$

7.10.5 Experiment to determine the specific heat capacity or a copper calorimeter

In order to determine the specific heat capacity of a substance, a calorimeter is used. The experiment is based on the law of conservation of energy.

Experiment 7.1

- Determine the mass of a clean, dry calorimeter.
- Fill it approximately one third full with cold tap water and determine the mass again.
- Wrap up the calorimeter with cotton wool and place it in a larger calorimeter.
- Heat a quantity of water, enough to fill the calorimeter completely.
- Take the temperature of the cold water in the calorimeter as well as the temperature of the hot water.
- Quickly but carefully pour the heated water into the cold water in the calorimeter while stirring it with a thermometer.
- Take the final temperature of the water in the calorimeter, which will also be the temperature of the calorimeter.
- Determine the mass again.

According to the law of conservation of energy, the heat emitted by the hot water must be equal to the heat absorbed by the calorimeter with cold water.

$$Heat\ released\ by\ hot\ water\ =\ mass\ of\ hot\ water\ \times\ c_\omega \times \Delta t$$
$$Heat\ absorbed\ by\ cold\ water\ =\ mass\ of\ cold\ water\ \times c_\omega \Delta t$$
$$Heat\ absorbed\ by\ copptr\ calorimeter\ =\ mass\ of\ copper\ \times c_\omega \times \Delta t$$

Now equate $heat\ absorbed\ =\ heat\ released$ and calculate the specific heat capacity c_k of the copper.

7.10.6 Water equivalent

The water equivalent (water value) of an object is the mass of water that has the same heat capacity as the object. Say a copper calorimeter has a mass of 100 g and the specific heat capacity of copper is 0,386 J fg.K.

If the temperature rises by 1 K, the copper absorbs heat energy equal to:

$Q = 100\ 0,386 \text{ x } 1$

$= 38{,}6 \text{ J}$

The heat capacity of the calorimeter is therefore 38,6 J/K.
But 9,22 g water also absorbs heat energy that is equal to:

$Q = 9{,}22 \text{ x } 4{,}187$

$= 38{,}6 \text{ J}$

Therefore the water equivalent of the copper calorimeter is 9,22 g, as the heat capacity of 9,22 g of water is equal to the heat capacity of 100 g copper.

From the above, it follows that:

water equivalent = mass of substance x specific heat capacity of substance + specific heat capacity of water.

or: $m_{water} x\, c_{water} = m_{subtance} x\, c_{substance}$ *where* m_{water} *= water equivalent.*

Worked Example 7.15

45g water at a temperature of 50 °C is poured into a calorimeter containing 42g of water at 20 °C. The final temperature of the mixture is 23 °C.

Calculate:

(a) the water equivalent of the calorimeter
(b) the mass of the calorimeter if it is made of brass having a specific heat capacity of 0,363 kJ/kg. °C.

Solution:

Given: 45 g water at 50 °C; 42 g water at 20 °C final temperature = 23 °C

(a) Heat released by 45 g water

$= m \text{ x } c \text{ x } (t_1 - t_3)$

$= 0{,}045 \text{ x } 4{,}187 \text{ x } (50 - 23)$

$= 5{,}087 \text{ kJ}$

Heat absorbed by 43 g water

$= m \text{ x } c \text{ x } (t_1 - t_3)$

$= 0{,}042 \text{ x } 4{,}187 \text{ x } (23 - 20)$

$= 0{,}528 \text{ kJ}$

Heat absorbed by water equivalent of calorimeter

$= m \text{ x } 4{,}187 \text{ x } (23 - 20) = 12{,}561 \text{ x m kJ}$

Heat released = heat absorbed

$5{,}087 = 0{,}528 + 12{,}561 \text{ x } m$

$m = 0{,}363 \text{ kg}$

(b) Water equivalent = mass of body x specific heat capacity

$0{,}363 = m \text{ x } 0{,}363$

$m = 1 \text{ kg}$

Worked Example 7.16

A steel block that has a mass of 0,16 kg is placed in an oven until it has the same temperature as the oven. It is then quickly transferred to a container that has a water equivalent of 50 g containing 1 kg of water. The temperature of the container rises from 20 °C to 24 °C and it is assumed that no heat is lost.

Calculate:

(a) the amount of heat transferred to the container and the water

(b) the temperature of the oven. The specific heat capacity of the steel is 500J/kg. °C.

Solution:

Given: *mo = 0,16 kg; heat equivalent of container =* m_c *= 50 g;* m_w *= 1 kg;* t_2 *= 24 °C;* t_1 *= 20 °C; co = 500 J/kg.°C; Q = ?; toven =?*

(a) heat absorbed by container and 1 kg water

= (mw + mc) x cw, x (t2 – t2)

= (1 + 0,05) x 4,187 x (24 - 20)

=17,585 kJ

(b) heat released by steel = heat· absorbed by water and container

$$m_o \times c_o \times (t_{oven} - 24) = 17{,}585$$

$$0{,}16 \times 0{,}5 \times (t_{oven} - 24) = 17{,}585$$

$$t_{oven} - 24 = 219{,}8$$

$$t_{oven} = 243{,}8\ °C$$

7.11 Important formulae for conservation of energy

- $Q = m \times c \times \Delta t$
- Heat released = heat absorbed
- Heat capacity = mass x specific heat capacity
  $C = m \times c$
- Water equivalent = mass of substance x specific heat capacity of substance $\div c$ of water.

Activity 7.1

1.

(a) Explain the difference between heat and temperature.

(b) Give the relation between degrees Celsius and kelvin.

(c) Convert 30 °C to kelvin. [303 k]

(d) Convert 196 K to degrees Celsius. [-77 °C]

2.

(a) A brass rod is heated from 22 °C to 120 °C. Calculate the increase in length if it is 2,4m long at 22 °C. The linear expansion coefficient of brass is 20 x 10^{-6}/°C. [4, 704 x 10^{-3} m]

(b) A copper pipe is 16 m long at a temperature of 19 °C. Steam at a temperature of 145 °C flows through the pipe. Calculate the extension of the pipe if the coefficient of linear expansion of copper is 17 x 10^{-6}/°C. [0,034]

3.

(a) The length of a rod increases from 1,041 8m to 1,043 7m when the temperature increases by 104 °C. Calculate the value of the linear expansion coefficient. [17, 54 x 10^{-6}/°C]

(b) A 1 m brass gauge rod measures correctly at 273 K. Calculate the error at 373 K. The linear expansion coefficient is 18 x 10^{-6} /K. [1,8 mm]

4. A steel plug has a diameter of 100 mm at 303 K. Calculate the temperature at which it will fit exactly into a hole with a constant diameter of 99,97 mm.
The linear expansion coefficient for steel is 11 x 10^{-6} /K-1. [275,5 K]
5. A steel tape measures the length of a copper rod as 900 mm when the temperature is 283 K. Calculate the reading at 308 K. The linear expansion coefficients for copper and steel are respectively 17 x 10^{-6} /K-1 and 11 x 10^{-6} /K. Assume that the steel tape reads correctly at 283 K. [900,135 mm]
6. A steel ball has a diameter of 60 mm and is 0,010 mm too large to pass through the hole in a brass plate when the ball and the plate are at a temperature of 30 °C. Calculate the common temperature for the ball and plate at which the ball will just pass through the hole. The linear coefficient of expansion of steel is 12 x 10^{-6} JK and that of brass is 19 x 10^{-6}/K. [327 K]
7.

(a) What is the relation between linear coefficient of expansion and area coefficient of expansion?

(b) A square copper plate has an area of 7 cm^2 at a temperature of 143°C.

Calculate:

(i) the side length of the plate at a temperature of 30 °C, if the linear expansion coefficient of copper is 18 x 10^{-6} rc (26,40 mm]

(ii) the change in area. [2,85 mm^2]

8. An iron plate is 10 mm square at 10 °C. The linear expansion coefficient of iron is 11 x 10^{-6}/°C.

Calculate:

(a) the area expansion coefficient of iron (22 x 10^{-6}/°C]

(b) the lengths of the sides at 31 °C [10, 002 13 mm]

(c) the new area at 31 °C. [100,046 2 mm^2]

9. A circular brass plate has an area 3,142 cm^2 at a temperature of 0 °C. The area increases by 5, 655 6 x 10^{-3} cm^2 when it is heated to 50 °C.

Calculate:

(a) the area expansion coefficient [36 x 10^{-6} /°C]

(b) the linear expansion coefficient [18 x 10^{-6} /°C]

(c) the amount by which the diameter will increase when the temperature rises from 0°C to 100 °C. [0,003 6 cm]

10. A metal plate is 1 m square at 283 K. It has a hole 600 mm in diameter in the centre. The linear coefficient of expansion of the metal is 12,5 x 10^{-6} /K-
1.

Calculate:

(a) the temperature of the metal plate if it is heated until the sides are 1,005m long. [684 K]

(b) the diameter of the hole at this temperature [603 mm]

(c) the increase in area at this temperature. [7 190,5 mm^2]

11.

(a) What is the relation between the coefficients of linear, area, and cubic expansion?

(b) The temperature of a copper sphere, diameter 48 mm, is increased by 164 °C. The linear expansion coefficient of copper is 17 x 10^{-6}/°C.

Calculate:

(i) the volume expansion coefficient [51 x 10^{-6}/°C]

(ii) the increase in volume of the sphere. (484,324 mm^3]

12. The sides of a brass cube measure 100 mm at 0 °C. Calculate the volume of the cube at 100 °C. The linear expansion coefficient of brass is 18 x 10^{-6}/°C. (1 005,4 cm^3]
13. A round copper bar has a diameter of 50 mm and is 1 m long. It is subjected to an increase in temperature of 95 °C. The linear expansion coefficient of copper is 17 x 10^{-6}/°C.

Calculate:
(a) the final length of the bar [1,001 62 m]
(b) the increase in surface area of the curved section (507,367 mm^2]
(c) the increase in volume [9 513,135 mm^3]
(d) the increase in surface area of each of the circular areas at the ends. [6,342 mm^2]
14.
(a) A metal ball has a volume of 0,95 m^3. The cubic coefficient of expansion is 36×10^{-6} /K-1

Calculate:
(i) the increase in volume if the temperature of the ball is increased by 200 K [6, 84 x 10^{-3} m]
(ii) the increase in diameter if the temperature is increased by 200 K. [2,927 mm]

(b) A steel block that measures 600 x 400 x 200 mm is heated from 20 °C to 220 °C. The linear coefficient of expansion of steel is 12×10^{-6} /K.

Calculate:
(i) the expansion in length of the longest side [1,44 mm]
(ii) the final area of the largest surface [0,241 m^2]
(iii) the increase in volume in m^3 [3,456 x 10^{-4} m^3]

Activity 7.2

1.
(a) Distinguish between absolute and apparent expansion of liquids.
(b) A measuring flask has a capacity of 50 cm^3 at 10 °C. Calculate the capacity of the flask at a temperature of 30 °C if the glass has a linear coefficient of expansion of 9×10^{-6} /°C. [50,027 cm^3]
(c) If the flask is completely filled with turpentine at 10 °C, calculate the amount of liquid that will overflow if the flask and its contents are heated to 30 °C. The cubic coefficient of expansion of turpentine is 97×10^{-5} /°C. [0,943 cm^3]

2. Calculate the increase in volume of 0,1 ***l*** mercury when the temperature changes from 283 K to 308 K. The cubic coefficient of expansion of mercury is 0,000 18 K^{-1}. [0,45 cm^3]
3. A glass vessel is completely filled with 80 cm^3 of mercury at 20 °C. The temperature of the glass vessel and its content increases to 40 °C. The linear coefficient of expansion of glass is 9×10^{-6}/K^{-1}, and the volumetric expansion of mercury is 180×10^{-6} /K^{-1}.

Calculate:
(a) the volume of the glass vessel [80,043 2 cm^3]
(b) the absolute expansion of mercury [0,288 cm^3]
(c) the apparent expansion of the mercury. [0,244 8 cm^3]

4. 100 ml ethyl alcohol is heated in a glass flask wit h a linear expansion coefficient of 9×10^{-6} /°C, from -15 °C to 40 °C, and in the process, 6,050 ml ethyl alcohol overflows because the flask was completely filled at -15 °C.

Calculate:
(a) the actual cubic expansion coefficient of the ethyl alcohol [1,127 x 10^{-3}/°C]
(b) the apparent cubic expansion coefficient of ethyl alcohol. [1,100 x 10^{-3}/°C]

5. A glass vessel with a capacity of 150 ml is completely filled with turpentine at 5 °C. The cubic expansion coefficient of glass is $24,8 \times 10^{-6}$ /°C. The vessel and turpentine are heated to 105 °C. If the apparent expansion coefficient of turpentine is $91,8 \times 10^{-6}$ °C, calculate the absolute expansion coefficient of the turpentine. [11, 66 x 10^{-3} /°C]
6.
(a) Why does a container filled with water overflow when it is heated?
(b) Describe an experiment that proves that a glass vessel becomes larger when heated.

(c) The temperature of a certain amount of water is 4 °C. Describe what happens to the volume and density of the water when it is:
(i) heated
(ii) cooled
(d) Why do water pipes sometimes crack during winter?

Activity 7.3

1.
(a) What do you understand by the term "thermodynamic temperature"?
(b) Convert 196 K to degrees Celsius and -10 °C to kelvin. [-77 °C; 263 K]
(c) A steel vessel contains carbon dioxide at 273 K and an absolute pressure of 1 200kPa. Calculate the internal gas pressure if the vessel is heated to 373 K. [1640 kPa]
2.
(a) Is the coefficient of expansion the same for all gases? Explain.
(b) Nitrogen having a volume of 3,6 m^3 at a pressure of 130 kPa is compressed to a pressure of 600 kPa while the temperature is increased from 18 °C to 178 °C. Calculate the final volume of the nitrogen. [1,209m^3]
(c) State Boyle's gas law.
3.
(a) Air is compressed in a cylinder from a pressure of 100 kPa and temperature of 20 °C so that t he volume occupies one quarter of the original volume while the temperature remains constant.

Calculate:
- (i) the final pressure of the air [400 kPa]
- (ii) the final temperature if the air is now heated to the original volume at the new pressure. [1 172 K]

(b) Compressed air at a pressure of 350 kPa and a temperature of 290 K flows through a heater. The temperature rises to 348 K and the pressure drops to 270 kPa. Calculate the percentage change in volume. [64,3 %]
4.
(a) State Charles's gas law.
(b) Oxygen at a pressure of 300 kPa and a temperature of 11 °C increases in volume from 0 , 025 m^3 while the temperature drops to 2 °C.

Calculate:
- (i) the final pressure of the oxygen [16,138 kPa]
- (ii) the volume that the oxygen would occupy at this final pressure if the temperature were kept constant at 11 °C. [0,465 m^3]

5.
(a) A gas at 273 K expands by 0,1 m^3 if it is heated through 50 K. Calculate the original volume at constant pressure. [0,546 m^3]
(b) Name the relation between volume, pressure, and thermodynamic temperature when the following are applied :
- (i) Boyle's law
- (ii) Charles's law
- (iii) the pressure law.

(c) A quantity of hydrogen is confined in a platinum chamber of constant volume. When the chamber is immersed in a container of melting ice, the pressure of the gas in the chamber is 100 kPa.

Calculate:
- (i) the temperature if the pressure gauge registered exactly 10 kPa [27,3 K]
- (ii) the pressure registered if the temperature of the chamber is increased to 100°C. (137 kPa]

6. A cylinder contains 0,11 m^3 nitrogen at a pressure of 1 700 kPa and a temperature of 284 K. The temperature decreases to 275 K while the volume remains constant.

Calculate:

(a) the pressure at this new temperature [1 464 kPa]

(b) the pressure if the volume changes to 0,095 m^3 while the temperature changes to 275 K. [1 906 kPa]

7. A container has a capacity of 0,08 m^3 and is filled with oxygen at a pressure of 400kPa. The temperature is 47 °C. Later it is found that, owing to a leak, the pressure has dropped to 320 kPa while the temperature has fallen to 27°C.

Calculate:

(a) the mass of oxygen that was initially in the container. Take R for oxygen as 260 J/kg.K [0,385 kg]

(b) the amount of oxygen that leaked out. [0,057 kg]

8. An air bubble at the bottom of a lake occupies 1 m^3 at a temperature of 7°C and a pressure of 450 kPa. The bubble rises to the surface, where thepressure is 101,3 kPa and the temperature is 27 °C. Calculate the size of the air bubble in m^3 when it reaches the surface. [4,76 m^3]

9. Gas with a volume of 2,8 m^3 at a pressure of 150 kPa is compressed to a pressure of 500 kPa.

Calculate:

(a) the final volume if the temperature increases from 20 °C to 170 °C [1,27m^3]

(b) the final volume if the temperature is kept constant at 20 °C. [0,84 m^3]

10. Helium at a pressure of 300 kPa and a temperature of 27 °C occupies 0,001m^3. It is heated until both the pressure and the volume double. The gas constant for helium is 2,08 kJ/kg.K.

Calculate:

(a) the final temperature [1 200 K]

(b) the mass of helium in grams. [0,481 grams]

11. A room measures 9 x 4 x 3, 2 m high and has a temperature of 20 °C at an atmospheric pressure of 96 kPa. The mass of 1 m 3 dry air at STP is 1,29 kg. Take R = 287,5 J/kg.K.

Calculate:

(a) the volume of 1 kg dry air in the room [0,877 m^3]

(b) the mass of dry air in the room. [131,36 kg]

12. The stroke length of a single-cylinder air compressor is 150 mm and the cylinder diameter is 100 mm. The free volume of the cylinder when the piston is at top dead centre is $\frac{1}{7}$ of the volume when the piston is at bottom dead centre. The pressure during the inlet stroke is 101,3 kPa, and the temperature of the inlet air is 27 °C. Calculate the final pressure during the compression stroke if the temperature rises to 75 °C. [822,556 kPa]

13. Two grams of nitrogen at 27 °C occupies a volume of 0,002 m^3.

Calculate:

(a) the pressure if the gas constant for nitrogen is 297 J /kg.K [89,1 kPa]

(b) the final volume if the pressure doubles and the temperature increases to 125 °C. [1, 327 x 10^{-3} m^3]

Activity 7.4

Take the heat capacity of water as 4,187 kJ/kg.K

1.

(a) What is meant by heat capacity of a substance and in what units is it measured?

(b) Calculate the heat capacity of a body if it is kept at a temperature of 200 K while it contains 50 kJ heat energy. [250J/k]

(c) What is meant by specific heat capacity of an object and in what units is it measured?

(d) Calculate the specific heat capacity of a body if it has a heat capacity of 250 J/K and a mass of 5 kg. [50 J/kg.K]

2.

(a) What factors influence the amount of heat energy absorbed by a substance?

(b) Water at a temperature of 380 K is pumped through a cooler and leaves the cooler at 367 K. If the heat energy released is 207,37 kJ/min, calculate the amount of water in *l*/min that flows through the cooler (1 litre water = 1 kg). [3,81 *l*/min]

3. A casting, mass 50 kg, is cooled down and loses 12,75 MJ heat energy. If the final temperature is 25 °C and the specific heat capacity of the metal is 0,377 kJ /kg. °C, calculate the initial temperature. [701,4 °C]

4.

(a) State the law of conservation of energy. What happens to the kinetic energy of a moving vehicle when the brakes are applied?

(b) Steel with a mass of 10 kg and a temperature of 150 °C is placed in water at a temperature of 30 °C. Assume there is no loss of heat. The specific heat capacity of steel is 0,47 kJ/kg.°C.

Calculate:

(i) the heat released by the steel if the final temperature is 40 °C [517kJ]

(ii) the volume of water used. [12,35*l*]

5. A wrought-iron bar having a mass of 60 kg is heated from 25 °C to 550 °C during heat treatment.

(a) Calculate the amount of heat that the bar must receive if the specific heat capacity of wrought iron is 460 J/kg.°C. [14,49 MJ]

(b) If this wrought-iron bar is now tempered in oil with a density of 0,8 kg/*l*, and the temperature increase of the oil may not exceed 27 °C, calculate the quantity of oil needed. The specific heat capacity of oil is 15 kJ/kg °C. [44,722*l*]

6. Calculate the amount of water, in litres, required to cool 20 steel parts if 10 per cent of the heat energy is lost and the temperature of the water may not rise by more than 70 °C. Mass of each part = 200 g Initial temperature of water = 15 °C Initial temperature of parts = 820 °C Specific heat capacity of steel = 460 J /kg. °C. [4,15 *l*]

7. Calculate the water equivalent of the following:

(a) 10 g lead, specific heat capacity 130 kJ /kg. °C. [0,31 kg]

(b) A calorimeter, mass 60 g and specific heat capacity 398 kJ/kg.K. [5,7kg]

(c) 1 kg aluminium, heat capacity 0,896 kJ/K. [0,214 kg]

8.

(a) Calculate the specific heat capacity of a calorimeter, water equivalent 912 g and mass 100 g. [0,385 kJ/kg.K]

(b) A piece of metal releases 628 J when its temperature drops from 100 °C to 30 °C.

Calculate:

(i) the water equivalent of the metal [2,14 g]

(ii) the heat capacity of the metal. [8,97 J/°C]

9. A steel bucket has a mass of 2 kg and contains 10 kg water at a temperature of 6 °C. The specific heat capacity of the steel is 500 J /kg. °C. 3 kg water at a temperature of 96 °C is now added.

Calculate:

(a) the water equivalent of the bucket and t he 10 kg water [10,239 kg]
(b) the final temperature of the bucket with its final content [26;4 °C]
(c) the heat capacity of the bucket with its final content [55,43 kJ/K]
(d) the heat energy of the bucket and its final content. [1,463 MJ]

10. The temperature of 0,3 kg aluminium is increased to 475 °C before the metal is immersed in 1,5 kg water at 10 °C in a copper calorimeter. The mass of the calorimeter is 0,25 kg and the specific heat capacity of copper is 390 J /kg °C. The final temperature of the calorimeter and its content is 30 °C.

Calculate:

(a)the water equivalent of the copper calorimeter [0,023 3 kg]
(b)the increase in the amount of heat of the calorimeter and the water that it contains [127,56 kJ]
(c) the specific heat capacity of the aluminium. [0,956 kJ/kg. °C]

11. A copper nugget with a mass of 180 g is heated and then immersed in 0,3 ***l*** water at a temperature of 287 K. The container has a water equivalent of 16 g. The final temperature is 297 K. Take the specific heat capacity of water as 4,2 kJ/kg.K and of copper as 390 J /kg.K.

Calculate:

(a) the amount of heat transferred to the water and the container [13,272kJ]
(b) the initial temperature of the copper [486,06 K]
(c) the heat capacity of the copper nugget before it is immersed. [0,070 2kJ /K]

12. A brass cylinder of mass 715 g is carefully immersed in water in a container. The container with water is equivalent to a mass of 630 g water. The temperature of the container and water rises from 15 °C to 28 °C, while the temperature of the brass cylinder decreases from 145 °C to 28 °C.

Calculate:

(a) the amount of heat transferred to the container and water [34,29 kJ]
(b) the specific heat capacity of the brass. [0,410 kJ /kg.°C]

13. A copper sphere of mass 8 kg is lowered into a container of water. The container with the water is equivalent to a mass of 7 kg water at 16 °C. The temperature increases to 35 °C. Assume that no heat is lost and that the specific heat capacity of copper is 390 J /kg.°C.

Calculate:

(a) the amount of heat transferred to the water and container [556,87 kJ]
(b) the initial temperature of the copper sphere. [213,5 °CJ

Self Check

I am able to:	YES	NO
• Change from Celsius to the kelvin temperature scale and vice versa.	○	○
• Analytically solve practical problems on the expansion of solids due to temperature changes (area and volume expansion).	○	○
• Analytically solve practical problems on volumetric expansion of liquids as a result of a rise in temperature.	○	○
• Be aware of the anomaly in the expansion of water.	○	○
• Reproduce the laws of Boyle and Charles in the form of specified mathematical equations.	○	○
• Do calculations which entail applications of the laws of Boyle and Charles and the combination of the two laws.	○	○

If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Past Examination Papers

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2013

NATIONAL CERTIFICATE

ENGINEERING SCIENCE N4

(15070434)

27 March 2013 (X-Paper)
09:00 – 12:00

TIME: 3 HOURS
MARKS: 100

Calculators may be used

This question paper consists of 6 pages and a 1-page formula sheet.

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Subsections of questions should be kept together. Draw a line after each question.
5. ALL formulae should be shown in the answer. Show all the steps in-between your answers.
6. Answers should be in blue or black ink.
7. ALL the sketches and diagrams should be done in pencil in the ANSWERBOOK.
8. Take g = 9,8 m/s^2
9. Write neatly and legibly

QUESTION 1

1.1 A small passenger boat, the 'Sunshine', leaves Cape Town harbour at a velocity of 80 km/h in a north-westerly direction. A freight ship, 'The Carrier', leaves Cape Town harbour simultaneously at a velocity of 50 km/h in a direction west 30° south.

Calculate the velocity (in magnitude and direction) of the 'Sunshine' relative to the velocity of 'The Carrier'. (5)

1.2 A canoeist is rowing on the Vaal dam at a velocity of 4 m/s in a northerly direction. A wind of 3 m/s suddenly starts blowing in a south-easterly direction.

Calculate the resultant velocity of the canoe in magnitude and direction. (5)

1.3 A soccer player kicks a ball at an angle of 20° to the horizontal. The initial velocity of the ball is 20 m/s. The soccer ball travels the path of a projectile. Calculate the following: (2)

1.3.1 The time taken by the ball to reach the maximum height (2)

1.3.2 The maximum height reached by the ball (2)

1.3.3 The horizontal displacement of the ball (3)

[17]

QUESTION 2

2.1 Explain the difference between linear velocity and angular velocity. (2)

2.2 A point on the circumference of a wheel of a locomotive has a velocity of 108 km/h. The diameter of the wheel is 80 cm. The velocity of the point on the wheel increases to 126 km/h during 50 seconds.
Calculate the following:

2.2.1 The initial and final angular velocities of the point on the wheel in rad/s (2)

2.2.2 The angular acceleration of the wheel (2)

2.2.3 The angular displacement of the wheel in radians during the Acceleration (2)

2.3 A force of 50 N is applied to the end of a door handle in order to open the door. The effective length of the door handle is 15 cm.
Calculate the following:

2.3.1 The torque on the door handle (2)

2.3.2 The work done on the door handle when it is turned through an angle of 45° (3)

[13]

QUESTION 3

3.1 Define Newton's second law of motion. (2)

3.2 A tractor is pulling a trailer with a mass of 4 ton up a hill with an incline of 5° at a constant velocity of 36 km/h. The trailer is experiencing a tractive resistance of 6 000 N.

Calculate the power required by the engine of the tractor if the efficiency of the engine is only 75%. (5)

3.3 A cyclist is travelling on a horizontal road at 10 m/s when noticing a stop sign 100 metres ahead. The brakes are applied and the bicycle comes to rest next to the stop sign. The resistance against motion is 5 newtons and the mass of the cyclist and bicycle is 75 kg.
Calculate the following:

3.3.1 The acceleration of the cyclist (2)

3.3.2 The total breaking force required to bring the bicycle to a standstill (3)

[12]

QUESTION 4

A light, horizontal beam ABCDE with A at the .left-hand side is 12 metres long. It Is supported at two points, A and D. A and Dare 10 metres apart. At 8, 5 metres from A, is a concentrated load of 10 kN. A concentrated .load of 6 kN is at C, 4 metres to the right of B. A concentrated load of 5 kN is at E, at the right end of the beam. There is a uniformly distributed load of 5 kN/m between points A and B.

4.1 Make a neat, labelled diagram of the beam as described above.
(HINT: Draw this diagram on a separate page and draw the diagrams in QUESTION 4.4 and QUESTION 4.6 below the diagram.) (10)

4.2 Calculate the reactions of the supports at points A and D and test your answers. (5)

[20]

4.3 Calculate the bending-moments at B, C, D and at a point, F, halfway between A and B.

4.4 Draw a shear-force diagram and show ALL the main values on the diagram.

4.5 Calculate the magnitude of the maximum bending moment.

4.6 Draw a neat bending-moment diagram and show ALL the main values on the diagram.

[16]

QUESTION 5

5.1 Define the unit pascal. (1)

5.2 The plungers of a three-cylinder, single-acting pump have diameters of 6 cm each and stroke lengths of 20 cm each. The pressure during the delivery stroke is 1,2 MPa.

Calculate the following:

5.2.1 The power required to drive the pump at 150 r/min if the efficiency of the motor is only 75% (5)

5.2.2 The volume of water delivered per minute if there is a 2% slip (3)

5.3 A simple-acting hydraulic jack with a lever is used to lift car engines in a workshop. The information below refers to the jack:

Diameter of ram cylinder	= 100mm
Stroke length of the plunge	= 40mm
Diameter of the plunger	= 20mm
Mechanical advantage of the lever	= 15

Calculate the following:

5.3.1 The effort that must be applied to the handle to lift an engine of 1 ton if the efficiency of the handle· is only 80% · (3)

5.3.2 The number of strokes needed by the plunger to lift the load by 500 mm if there is a slip of 6% (2)

[15]

QUESTION 6

6.1 Name TWO types of stresses found in materials. (2)

6.2 A steel pillar with a diameter of 40 cm is used to support part of a balcony. The pillar is subjected to a stress of 8 MPa. (3)

Calculate the maximum load allowed on the steel pillar.

6.3 A square cast-iron bar is placed under 50 kN pressure.

6.3.1 Calculate the dimensions of the bar if the stress in the bar is not to exceed 300 kN. (3)

6.3.2 If the bar has a length of 20 cm before the load is applied, and a final length of 20,002 cm after the load is applied, then calculate the strain in the bar. (2)

6.4 Write down the positions of the following:

6.4.1 The centroid of a thin circular plate with a radius r, resting on a point on its circumference (1)

6.4.2 The centre of gravity of a solid cube with a volume of $V = P$ (2)

[13]

QUESTION 7

7.1 Define Boyle's law and write down an equation to correspond with this law. (2)

7.2 Define Charles' law and write down an equation to correspond with this law. (2)

7.3 A solid steel ball with a volume of 50 cm^3 is at a temperature of 20 °C and has to increase to 50,6 cm^3 to be able to fit through a hole.

Calculate to which temperature the ball has to be heated to in order to fit through the hole. The steel has a linear coe3fficient of expansion of 12×10^{-6} /K. (3)

7.4 A given mass of chlorine gas has a volume of 40 cm^3 at 20 ºC. Calculate its volume at 50 ºC if the pressure remains constant. (3)

7.5 The density of oxygen is 1,42 kg/m^3 at standard temperature and pressure (STP). Calculate the density of oxygen at 30 ºC and a pressure of $0,9 \times 10^5$ Pa.
The gas constant, R for oxygen is 261,311J/kgK. (4)

[14]

TOTAL: 100

ENGINEERING SCIENCE N4

FORMULA SHEET

Any applicable formula may also be used.

$S = \frac{u + v}{2} \times t$	$a = \alpha R$	$H.V. = \frac{F_p}{F_h} = M.A.$
$\overline{V} = \frac{s}{t}$	$v = \pi DN$	$AV = mgh = WD$
$v = u + at$	$T = FR$	$\underline{Q} = mc\Delta t$
$s = ut + \frac{1}{2} as^2$	$AV = T\theta = WD$	$\Delta l = lo\alpha\Delta t$
$v^2 = u^2 + 2as$	$P = 2\pi NT$	$\beta = 2\alpha$
$v_g = \frac{u + v}{2}$	$P = Fv$	$\gamma = 3\alpha$
$\omega = 2\pi N$	$P = T\omega$	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
$\omega = \frac{\theta}{t}$	$F_\alpha = ma$	$PV = mRT$
$\theta = \frac{\omega_2 + \omega_1}{2} \times t$	$E_p = mgh$ –	$\in = \frac{x}{l}$
$\omega_2 = \omega_1 + \alpha t$	$E_k = \frac{1}{2} \mu\varpi^2$	$E_k = \frac{\sigma}{\in}$
$\theta = \omega_2 t + \frac{1}{2} \alpha t^2$	$P = \frac{F}{A}$	$\sigma = \frac{F}{A}$
$v = \omega R$	$m = \rho \times vol$	$E = \frac{Fl}{Ax}$
$\theta = 2\pi n$	$P = \rho gh$	$\overline{y} = \frac{A_1y_1 \pm A_2y_2 \ldots}{A_1 \pm A_2 \ldots}$
$S = R\theta$	$\frac{W_r}{F_p} = \frac{D^2}{d^2}$	$\overline{y} = \frac{v_1y_1 \pm v_2y_2 \ldots}{v_1 \pm v_2 \ldots}$
$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta}$	$W.D. = P \times V = A..V.$	

Marking Guidelines

higher education
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Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2013

NATIONAL CERTIFICATE

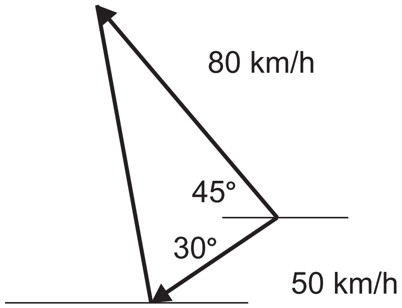
ENGINEERING SCIENCE N4

(**15070434**)

This marking guidelines consists of 10 pages.

QUESTION 1

1.1



$c^2 = a^2 + b^2 - 2ab\cos75°$
$= 802 + 502 - 2 \times 80 \times 50\cos75°$
$= 6829{,}445$
$C = 82{,}64$ km/h

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$\sin A = (a \sin C) + c$
$\sin A = (80 \sin 75°)/(82{,}64)$
$= 0{,}935$
$A = 69{,}24°$

Therefore Angle: 30° + 69,24°-90° = 9,24°

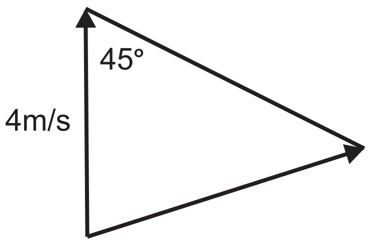
Alternative solution:

HC = 80 cos45°+50cos30°
= 81,569 km/h
VC = 8sin45°
C =82,64km/h
$R = \sqrt{(81{,}569^2) + (-13{,}267^2)}$
= 82,641*km/h*

The velocity of the "Sunshine" relative to "The Carrier" is 82,64km/h North 9,24°West

(5)

1.2



$c^2 = a^2 + b^2 - 2ab\cos45°$
$= {}^{4}2 + 32 - 2 \times 4 \times 3\cos45°$
$= 8{,}029$
$C = 2{,}834$ m/s

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$\sin B = (3 \sin45°) - 2{,}834$
$= 0{,}748$
$B = 48{,}463°$

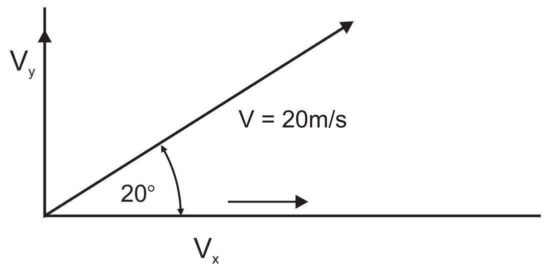
Alternative solution:

HC = 3cos45°
= 2,1213m/s
VC = 4-3sin45°
C = 1,8787m/s
$R = \sqrt{(2{,}1213^2) + (-1{,}8787^2)}$
= 2,834m/s

Resultant velocity of the canoeist is 2,834m/s North 48,463° East

(5)

1.3 (2)



Resolve the velocity into the vertical and horizontal components:
Vx = 20 x cos 20º = 18,794 m/s
Vv = 20 x sin 20 º = 6,84 m/s

1.3.1 v = u + at - therefore t = (v - u) + a (2)
= (0- 6,84) ÷ (-9,8)
= 0,698 seconds

1.3.2 s(height) = $(v^2 - u^2) + 2a$ (2)
= 0- $(6,84)^2$ + 2(-9,8) +2(9,8)
= 2,387 meter

1.3.3 s(horizontal) = V_x x t (3)
= 18,794 X 2 X 0,698
= 26,236 meter

QUESTION 2 **[17]**

2.1 Angular velocity is the rate of change of angular displacement about an axis and is measured in radians per second. (2)
Linear velocity is the rate of change of linear displacement and is measured in metres per second

2.2.1 $\omega 1$ = V/3,6 ÷ r
therefore $\omega 1$ = 108/3,6 ÷ 0,4 (2)
$\omega 1$ = 75 rad/s
$\omega 2$ = 126/3,6 ÷ 0,4
= 87,5 rad/s

2.2.2 $\alpha = (\omega_2 - \omega_1)/t$ (2)
= (87,5- 75)/50
= 0,25 radians/s^2

2.2.3 $\theta = \omega 1$t + 1/2 at2 (2)
= 75 x 50 + ½ x 0,25 x 50 x 50
= 3750 + 312,5 rad
= 4062,5 rad

2.3.1 T = F x r (2)
= 50 X 0,15
= 7,5 N.m

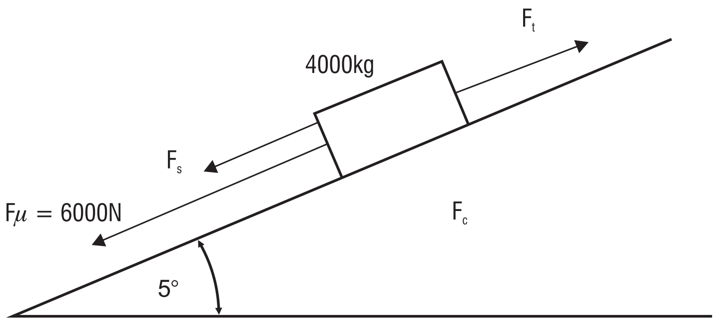
2.3.2 W = T x θ θ = 2 πn
= 7,5 x 0,785 = $2\pi \frac{45}{360}$ (3)
= 5,89 J = 0,785 rad

[13]

QUESTION 3

3.1 The change in momentum of a body is proportional to the force applied to the body and takes place in the direction of the applied force. (2)

3.2 (5)



F_s = mg x sinΘ

= 4000 x 9,8 x sin5°

= 3416,505N

F_{st} = F_s + $F\mu$

= 3416,505 + 6000

= 9416,505N

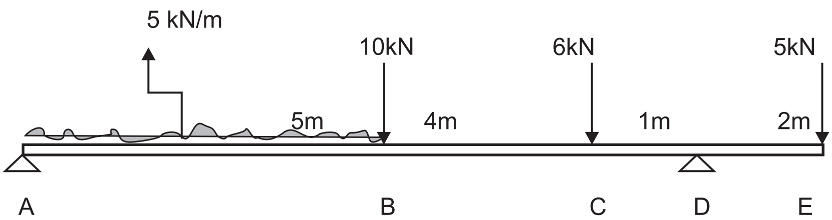
P_{st} = F x v
= 9416,505 x 36/3,6 x 100/75
= 125 553,4 W = 125,553 kW

3.3.1 a = $(v^2 - u^2) \div 2s$ (2)
= (0- 1 00)/2x1 00
= -0,5 m/s^2

3.3.2 F = ma - 5 (3)
= (75 x 0,5) - 5
= 32,5 N

QUESTION 4 **[12]**

4.1 (10)



4.2 Take moments about D: (5)
Σ CWM = Σ ACWM
(A x 10) + (5 x 2) = (25 x 7,5) + (10 x 5) + (6 x 1)

A = 23,35 kN

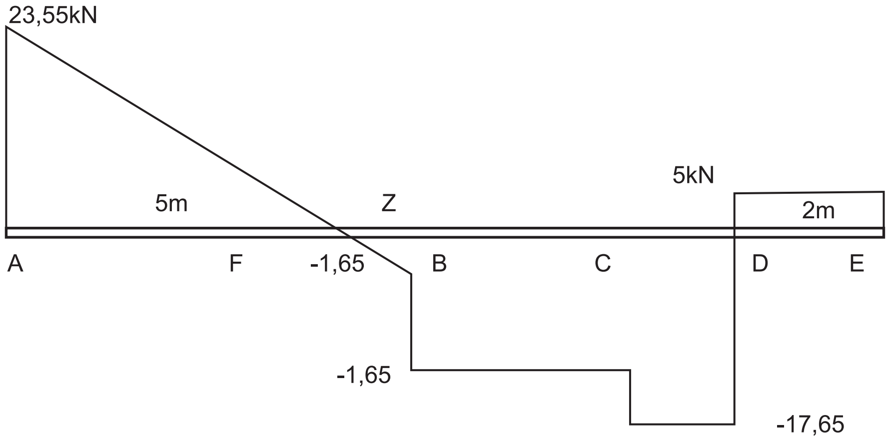
Take moments about A:
Σ ACWM = Σ CWM
(5 x 12) + (6 x 9) + (10 x 5) + (25 x 2,5) = Dx10
D = 22,65 kN

Test: Σ↑ F = 23,35 + 22,65 = 46 kN
And Σ↓ F = 25 + 10 + 6+5 = 46 kN

[20]

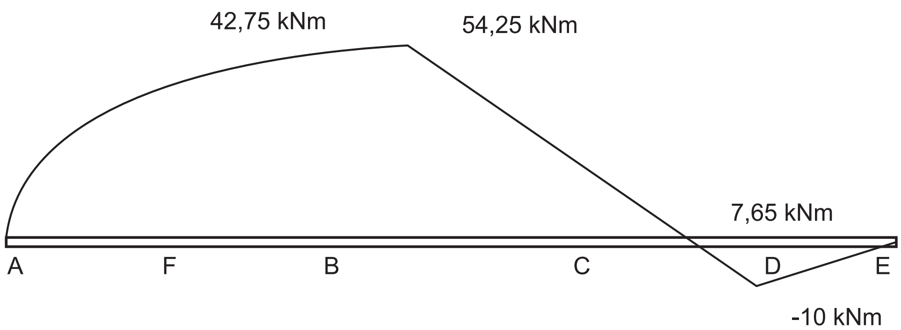
4.3 BM at A = 0
BM at 8 = -(5 x 7)- (6 x 4) + (22,65 x 5) = 54,25 kNm
BM at C = -(5 x 3) + (22,65 x 1) = 7,65 kNm
BM at D = -10 X 1 = -10 kNm
BM at F (point half way between A and B)
= -(5 x 9,5)- (6 x 6,5) - (10 x 2,5) + (22,65 x 7,5) - (5 x 2,5 x 1,25)
= 42,75 kNm

4.4



4.5 Let the maximum bending moment be at z
Shear forces at z = 0
Therefore- (5 x z) + 23,35 = 0
z = 4,67 metres
therefore maximum bending moment is 4,67 metres from the left
Maximum bending moment at z = (23,35 x 4,67)- (5 x 4,67 x 4,67/2)
= 54,523 kNm

4.6



[16]

QUESTION 5

5.1 The pressure exerted when a force of 1 Newton acts upon an area of 1 m2 is known as the Pascal. (1)

5.2.1 V = $\pi D^2/4$ x h x 3
= π x $(0{,}06)^2/4$ x 0,2 x 3
= 1,697 x 10^{-3} m^3
Power = Pressure x Volume
= 1,2 x 10^6 x 1,697 x 10^{-3} x 150/60 x 100/75
= 6788 Watt
= 6,788 kW (5)

5.2.2 Volume = 1 ,697 x 10^{-3} x speed of pump
= 1,697 x 10^{-3} x 150 x 98/100
= 0,25 m3
= 250 litres (5)

5.3.1 $F/D^2 = f/d^2$
f = F d^2/D^2
= 9800 x $0{,}02^2 \div 0{,}1^2$
=392 N

Effort on lever = (392 ÷ 15) x 100/80
= 32,667 N (3)

5.3.2 D^2 x H = n x d^2 x h
$(0{,}1)^2$ x 0,5 = n X $(0{,}02)^2$ x 0,04
n = 312,5

But there is a slip of 6% therefore n = 312,5 x 106/100 = 331,25 strokes (3)

[15]

QUESTION 6

6.1 Shear stress, compressive stress and tensile stress (2)

6.2 P = F/A
F= P x A

= 8 x 10^6 x $\pi(0{,}4)^2/4$
= 1005309,649 N
= 1005,31 kN (3)

6.3.1 P = F/A
A = F/P
= 50 000/300000
= 0,167 m^2 (3)

6.3.2 A = $l \times I$
$l = \sqrt{A}$
= 0,408 m = 40,8 cm
∈ = x/l = 0,002/20 = 0,0001 (2)

6.4.1 C = r (1)

6.4.2 $V = \beta \text{ therefore } G = \frac{1}{2} \text{ for all 3 dimensions}$ (2)

[13]

QUESTION 7

7.1 The volume of a given mass of gas is inversely proportional to the pressure, provided that the temperature is kept constant. Thus for a given mass of gas p x V = constant. Therefore $P_1V_1 = P_2V_2$ = constant (2)

7.2 The volume of a given mass of gas is directly proportional to its thermodynamic temperature, provided that the pressure is kept constant.
So for a given mass of gas V/T = constant and $V_1/T_1 = V_2/T_2$ (2)

7.3 $\Delta V = V_\sigma \times 3\alpha \times \Delta t$
0,6 = 50 x 3 x 12 x 10^{-6}
Δt = 333,333.ºC
t_f = 333,333 + 20 = 353,333 ºC (3)

7.4 $P_1 V_1/T_1 = P_2V_2/1\ T_2$ but the pressure remains constant
$V_1/T_1 = V_2/T_2$ (3)
40/293 = V_2/323
V_2 = 44,096 cm^3

7.5 STP implies that T = 273 Kelvin and P = 101,3 kPa
Density = 1 ,42 kg/m^3 implies that 1,42 kg of the gas has a volume of 1 m^3
Therefore $P_2V_2 = mRT_2$
0,9 x 10^5 x V_2 = 1,42 x 261,311 x 303- the mass remains constant
V_2 = 1,249 m^3
Density of oxygen = *mN* = 1,42/1,249 = 1,137 kg/m^3 (4)

[14]

TOTAL: 100

Past Examination Papers

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2012

NATIONAL CERTIFICATE

ENGINEERING SCIENCE N4

(**15070434**)

27 March 2012 (X-Paper)
09:00 – 12:00

This question paper consists of 6 pages and a 1-page formula sheet.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Subsections of questions should be kept together.
5. Rule off across the page on completion of each question.
6. ALL formulae should be shown in the answer. Show ALL the steps in between.
7. Questions should be answer in blue or black ink.
8. ALL the diagrams should be in pencil.
9. Take g = 9,8 m/s²
10. Write neatly and legibly

QUESTION 1

1.1 Two cars leave a fork in a road simultaneously. Car V travels at 60 km/h to the west and car W travels at 100 km/h in a direction N 45° E.

Calculate the relative velocity of car W with respect to car V in magnitude and direction. (5)

1.2 A cyclist is cycling at 8 m/s to his destination due north. There is a wind blowing at 3 m/s from the south-east. Calculate the resultant velocity of the cyclist in magnitude and direction. (5)

1.3 A bullet is fired from a rifle at an angle of 20° to the horizontal with an initial velocity of 250 m/s.

Calculate the following:

1.3.1 The time for the bullet to reach the maximum height (3)

1.3.2 The maximum height reached by the bullet (2)

1.3.3 The horizontal displacement of the bullet (2)

[17]

QUESTION 2

2.1 Define *angular acceleration.* (1)

2.2 The spindle of a washing machine rotates at 800 r/min and slows down to 300 r/min during 30 revolutions.

Calculate the following:

2.2.1 The angular displacement in radians (2)

2.2.2 The angular acceleration (3)

2.2.3 The time required to turn through the 30 revolutions (2)

2.3 The wheel of a locomotive has a diameter of 450 mm, and the locomotive is travelling at 90 km/h. Calculate the angular velocity of the wheel. (2)

[10]

QUESTION 3

3.1 The engine of a truck exerts a force of 70 kN on the truck as it travels up an incline of 1º. The truck experiences a resistance of 60 N per ton of the weight of the truck. The total mass of the truck and its engine is 240 ton.

Calculate the following:

3.1.1 The acceleration of the truck (4)

3.1.2 The braking force that would be required on the return journey to prevent the acceleration exceeding 0,02 m/s^2 (3)

3.2 Calculate the power required to pull a mass of 200 kg at a constant velocity of 4 m/s on a horizontal plane. The kinetic coefficient of friction is 0,8. (3)

[10]

QUESTION 4

4.1 A light horizontal beam ABCD, is 7,5 m long. The distance between A and B is 3 m, the distance between B and C is 3 m and the distance between C and D is 1,5 m. The beam uniformly distributed load of 60 kN/m between A and B, a uniformly distributed load 20 kN/m between B and C and a concentrated load of 100 kN at D. The beam is supported at A and C.

4.1.1 Draw a neat labelled diagram of the beam and then calculate the reaction forces at the supports. (4)

4.1.2 Calculate the bending moments at B, C, E (a point halfway between A and B) and F (a point halfway between B and C). (4)

4.1.3 Draw a shear force diagram and a bending moment diagram and indicate ALL the values on the diagrams. (6)

4.2 Define the term *centre* of *gravity.* (1)

[15]

QUESTION 5

5.1 Define *Pascal's Law.* (2)

5.2 The diameter of the ram piston of a press is 125 mm and that of the plunger is 25 mm. The mechanical advantage on the plunger is 16. The stroke length of the plunger is 50 mm.

Calculate the following:

5.2.1 The effort that has to be applied to the handle in order to lift a load of 500 kg (3)

5.2.2 The distance the load will be raised after 50 strokes of the plunger (3)

5.2.3 The work done by the press to lift the load of 500 kg after 50 strokes. The efficiency of the press is only 80%. (3)

5.3 The plungers of a three-cylinder pump have diameters of 75 mm and stroke lengths of 250 mm. The pressure during a delivery stroke is 750 kPa.

Calculate the power required to drive the pump at 150 r/min if the efficiency of the motor is 75%. (5)

5.4 A hydraulic accumulator has to deliver water at a pressure of 2,8 MPa. Calculate the mass of the ballast if the ram has a diameter of 300 mm. (3)

[19]

QUESTION 6

6.1 Define *Young's Modulus of Elasticity* of a material. (2)

6.2 An elastic rod is 5 m long and has a cross-sectional area of 1,5 cm². The rod hangs vertically and stretches with 0,075 cm when a mass of 350 kg is attached to its free end.

Calculate the following:

6.2.1 The stress (2)

6.2.2 The strain (2)

6.2.3 Young's Modulus of elasticity for the material (2)

6.3 A lift has a mass of 600 kg and is designed for a maximum upward acceleration of 3,2 m/s². If the cross-sectional area of the cable is 3 cm², and the maximum tension may not exceed 40 MPa, calculate the maximum load, in kilogram, that the lift may carry. (5)

[13]

QUESTION 7

7.1 Define the *coefficient of linear expansion* of a substance. (2)

7.2 The area of a brass plate is 1 cm² at 0 ºC and 1,004 cm² at 100 ºC. Calculate the coefficient of linear expansion of brass. (3)

7.3 A glass flask is completely filled with 100 ml alcohol and heated from 15 ºC to 40 ºC. 6,05 ml alcohol overflows in the process. The coefficient of linear expansion of glass is 9×10^{-6}/ºC. Calculate the coefficient of cubic expansion of alcohol. (5)

7.4 The volume of 2 grams of nitrogen at 27 ºC is 0,002 m³

Calculate the following:

7.4.1 The pressure of the gas if the gas constant for nitrogen is 297 J/kgK. (3)

7.4.2 The final volume if the pressure doubles and the temperature increases to 125 °C.

(3)

[16]

TOTAL: 100

ENGINEERING SCIENCE N4

FORMULA SHEET

Any applicable formula may also be used.

$S = \frac{u+v}{2} \times t$	$a = \alpha R$	$H.V. = \frac{F_p}{F_h} = M.A.$
$\overline{V} = \frac{s}{t}$	$v = \pi DN$	$AV = mgh = WD$
$v = u + at$	$T = FR$	$\underline{Q} = mc\Delta t$
$s = ut + \frac{1}{2} as^2$	$AV = T\theta = WD$	$\Delta l = lo\alpha\Delta t$
$v^2 = u^2 + 2as$	$P = 2\pi NT$	$\beta = 2\alpha$
$v_g = \frac{u+v}{2}$	$P = Fv$	$\gamma = 3\alpha$
$\omega = 2\pi N$	$P = T\omega$	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
$\omega = \frac{\theta}{t}$	$F_\alpha = ma$	$PV = mRT$
$\theta = \frac{\omega_2 + \omega_1}{2} \times t$	$E_p = mgh$	$\in = \frac{x}{l}$
$\omega_2 = \omega_1 + \alpha t$	$E_k = \frac{1}{2} \mu\varpi^2$	$E_k = \frac{\sigma}{\in}$
$\theta = \omega_2 t + \frac{1}{2} \alpha t^2$	$P = \frac{F}{A}$	$\sigma = \frac{F}{A}$
$v = \omega R$	$m = \rho \times vol$	$E = \frac{Fl}{Ax}$
$\theta = 2\pi n$	$P = \rho gh$	$\overline{y} = \frac{A_1y_1 \pm A_2y_2 \ldots}{A_1 \pm A_2 \ldots}$
$S = R\theta$	$\frac{W_r}{F_p} = \frac{D^2}{d^2}$	$\overline{y} = \frac{v_1y_1 \pm v_2y_2 \ldots}{v_1 \pm v_2 \ldots}$
$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta}$	$W.D. = P \times V = A..V.$	

Marking Guidelines

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2012

NATIONAL CERTIFICATE

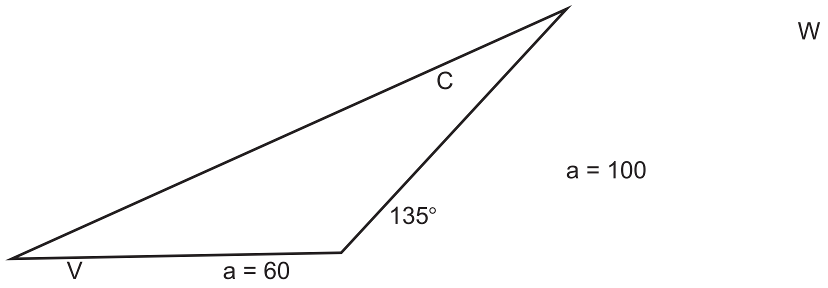
ENGINEERING SCIENCE N4

(15070434)

This marking guidelines consists of 9 pages.

QUESTION 1

1.1 (5)



$c^2 = a^2 - b^2 - 2ab\cos 135°$

$= 60^2 + 100^2 - 2 \times 60 \times 100\cos 135°$

$= 22085,3$

$C = 148,61$km/h

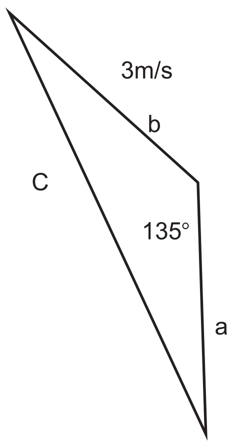
$\frac{\sin b}{b} = \frac{\sin c}{c}$

$\sin b = b \sin c) \div c$

$b = 28,41°$

Relative velocity of car W with respect to car V is 148,61 km/h east 28,41º North

1.2 (5)



$c^2 = a^2 - b^2 - 2ab\cos 135°$

$= 8^2 + 3^2 - 2 \times 8 \times 3\cos 135°$

$= 106,94$

$c = 10,34$m/s

$\frac{\sin b}{b} = \frac{\sin c}{c}$

$\sin b = b \times \frac{\sin c}{c}$

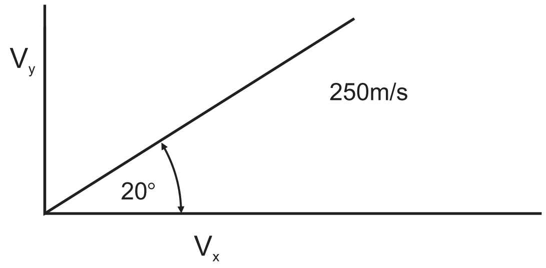
$= 3 \times \frac{\sin 135}{10,34}$

$= 0,205156$

$B = 11,84°$

Resultant velocity of the cyclist is 10,34 m/s N 11,84º W

1.3



Resolve the velocity into the vertical and horizontal components:
$V_x = 250\cos 20° = 234{,}9$m/s
$V_y = 250\sin 20° = 85{,}505$m/s

1.3.1 v = u + at – therefore t = (v - u) ÷ a
= (0 - 85,505) + (-9,8)
= 8,725 seconds (3)

1.3.2 height) = $(v^2 - u^2) \div 2a$ (2)
= 0 - $(85{,}505)^2$ +2(-9,8)
= 373 meter

1.3.3 s(horizontal) = V_x x t (2)
= 234,9 x 2 x 8,725
= 4099 m = 4,9 km

[17]

QUESTION 2

2.1 Angular acceleration is the rate of change of the angular velocity (1)

2.2.1 $\theta = 2\pi n$ (2)

= 2 x π x 30
= 188 radians

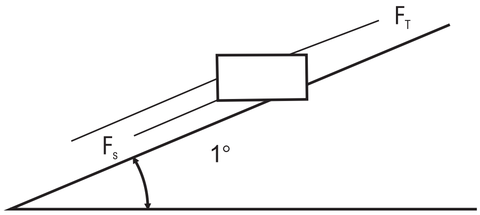
2.2.2 $\alpha = (\omega_2)^2 - (\omega_1)^2 / 2\,\theta$ (3)
= $\frac{(300\times 2\pi/60)2-(800\times 2\pi/60)2}{2\times 2\times \pi\times 30}$
= -16 rad/s^2

2.2.3 t = $(\omega_2 - \omega_1) / \alpha$ (2)
= (300 x 2π/60)- (800 x 2π/60) ÷ (-16)
= 3,27 sec

2.3 v = ωr (2)
ω = v/r = 90/(3,6 x 0,225) = 111,11 rad/s
[10]

QUESTION 3

3.1



3.1.1 $F_T = F_a + F_s + F_R$
$F_a = F_T - F_s - F_R$
$= 70\ 000 - mg\sin\theta - (60 \times 240)$
$= 70\ 000 – 41048 - 14\ 400$
$= 14\ 552$ N
$a = F/m$
$= 14\ 552/240\ 000$
$= 0{,}0606$ m/s^2 (4)

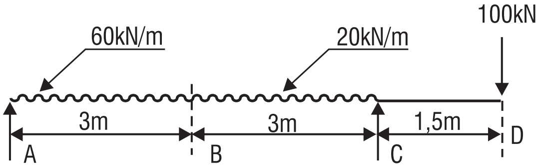
3.1.2 $Fa = Fs - (F_B + F_R)$
$ma = mg\sin\theta - (F_B + 14400)$
$240\ 000 \times 0{,}02 = 240\ 000 \times 9{,}8 \sin1° - (F_B + 14400)$ (3)
$F_B = 21\ 848$ N

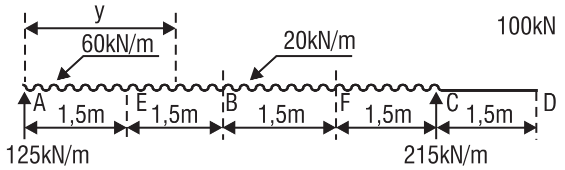
3.2 $P = W/t$
$= F \times s/t$ $\quad F = \mu \times W$
$= F \times v$ $\quad = 0{,}8 \times 200 \times 9{,}8 = 1568$ N
$= 1568 \times 4$ (3)
$= 6272$ Watt

[10]

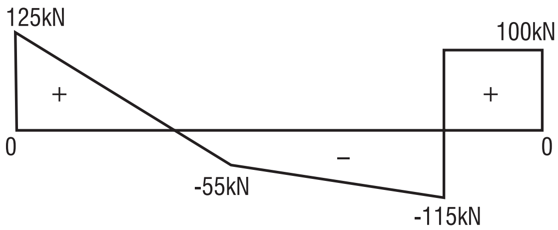
QUESTION 4

4.1

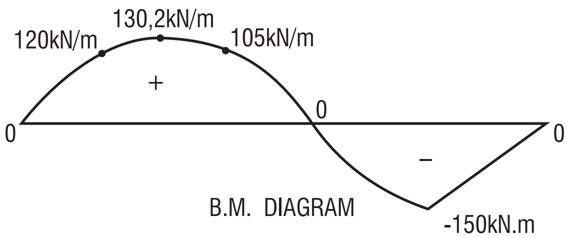




LOAD DIAGRAM



S.F. DIAGRAM



B.M. DIAGRAM

4.1.1 Take moments about C:
Σ CWM = Σ ACWM (4)
R_A x 6 + (100 x 1,5) = (180 x 4,5) + (60 x 1,5)
R_A = 125 kN

Take moments about A:
Σ ACWM = Σ CWM
R_c x 6 (180 x 1,5) + (60 x 4,5) + (100 x 7,5)
R_c = 215 kN

Test: Σ↑ F = 125 + 215 = 340 kN
And Σ↓ F = 180 + 60 + 100 = 340 kN

4.1.2 BM at A = O
BM at B = (125 x 3) - (180 x 1,5) = 105 kNm (4)
BM at C = -100 x 1,5 = -150 kNm
BM at D = 0
BM at E = (125 x 1,5) -60 x 1,5 x 1,5/2 = 120 kNm
BM at F = (215 x 1,5) - 20 x 1,5 x 1,5/2 -100 x 3 = 0

4.1.3 See attached - Not compulsory for candidate to determine position and magnitude of the maximum bending moment. (6)

4.2 The center of gravity is the average location of the weight of an object. (1)
OR
The center of gravity of a body is a point in space where, for the purpose of - various calculations, the entire mass of a body is concentrated

[15]

QUESTION 5

5.1 The pressure exerted on liquid acts on the liquid in all directions and with the same magnitude (2)

5.2.1 $\frac{f}{d^2} = \frac{F}{D^2}$ (3)

f = F d^2/D^2
= 500 x 9,8 x $25^2/125^2$
= 196 N
But MA = 16 so f = 196/16 = 12,25 N

5.2.2 D^2 H = nd^2h (3)
125^2 H = 50 x 25^2 x 50
H = 100 mm

5.2.3 Work done = F x s x 80/100
= 500 X 9,8 x 0,1 x 80/100 (3)
= 392 J

5.3 Volume per stroke = $\pi d^2/4$ x h x 3
= π x $(0{,}075)^2/4$ x 0,25 x 3
= 0,0033147 m^3
Pump runs at 150 rev/min = 150/60 = 2,5 rev/s
Volume per second= 0,0033147 x 2,5 = 0,00828 m^3
P = Work done/ time
= pressure x volume x 100/η
= 750 X 0,00828 X 100/75 = 8,283 kW (5)

5.4 Pressure = F/a
F = Pressure x a (3)
= p x $\pi d^2/4$
= 2,8 x 10^6 x π x $0{,}3^2/4$
= 198000 N
m = F/g = 198000/9,8-= 20204,08 kg = 20,204 ton

[19]

QUESTION 6

6.1 The ratio of stress to strain is a constant within the elastic limit, so that $\sigma\varepsilon$ is a constant for a specific material. This is called Young's Modulus of Elasticity. The modulus of elasticity is a measure of the stiffness of the material. (2)

6.2.1 $\sigma = F/a$ (2)
= 350 x 9,8/1,5 x 10^{-4}
= 22,867 MPa

6.2.2 $\in = \square/l = 0{,}075 \times 10^{-2}/5 = 1{,}5 \times 10^{-4}$ (2)

6.2.3 $E = \sigma/\in = 22{,}867 \times 10^{6} / 1{,}5 \times 10^{-4} = 152{,}44\ GPa$ (2)

6.3 Let □ be the load in kilogram that the lift may carry:
Force required upwards: F_{up} = mg ÷ ma
= (600 + □) x 9,8 ÷ (600 + □) x 3,2
σ = F_{up}/a = [(600 + □) x 9,8 ÷ (600 + □) x 3,2] ÷ 3 x 10^{-4} (5)
40 x 10^{6} x 3 x 10^{-4} = 5880 + 9,8 □□ + 1920 + 3,20 □
□ = 323 kg

[13]

QUESTION 7

7.1 The coefficient of linear expansion of a substance indicates the change in length per unit length per degree change in temperature and depends on the nature of the substance (2)

7.2 ΔA = 1,004 – 1 - 0,004 x 10^{-4} m^2
And ΔA = A_0 x 2α x Δt
0,004 x 10^{-4} = 1 x 10^{-4} x 2 x α x 100
α = 2 x 10^{-5} /C° (3)

7.3 ΔV_{glass} = V_o x 3 α x Δt $\quad$ V_o = 100 x litres
= 100 x 10^{-6} x 3 x 9 x 10^{-6} x 25
= 0,0675 ml
Volume alcohol overflowing due to expansion: 6,05 - 0,0675 = 5,9825 ml
$\Delta V_{alcohol}$ = V_o x 3 α x Δt
5,9825 X 10^{-6} = 100 x 3 α x 25 $\quad$ 3α = y
γ = 3 α = 2,393 x 10^{-9} /C° (5)

7.4.1 PV = mRT (3)
P = mRT/V
= 0,002 x 297 x 300/0,002
= 89,1 kPa

7.4.2 $P_1V_1/T_1 = P_2V_2/T_2$
89,1 x 0,002/300 = 178,2 X V_2/398
v_2 = 0,00132 m^3
(3)

[16]

TOTAL: 100

Past Examination Papers

higher education
& training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

AUGUST 2012

NATIONAL CERTIFICATE

ENGINEERING SCIENCE N4

(**15070434**)

27 July 2012 (X-Paper)
09:00 – 12:00

TIME: 3 HOURS
MARKS: 100

This question paper consists of 6 pages, 1 formula sheet and 1 information sheet

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
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6. Answers should be in blue or black ink.
7. ALL diagrams should be in pencil.
8. Take $g = 9{,}8\ m/s^2$
9. Write neatly and legibly

QUESTION 1

1.1 A light aircraft is flying at 250 km/h in calm weather. A wind is blowing from the north-west at 30 km/h. Calculate the direction in which the pilot has to steer in order to fly due south, as well as the resultant velocity of the aircraft in relation to the ground. (4)

1.2 A train is travelling in an easterly direction at 100 km/h. A passenger notices an aeroplane appearing to fly at 150 km/h north. Calculate the actual velocity and direction of the aeroplane. (4)

1.3 A stone is projected vertically upwards at a velocity of 50 m/s.

Calculate:

1.3.1 The time it takes the stone to reach the ground again (2)

1.3.2 The maximum height reached by the stone (2)

1.4 A bullet is fired at such an angle that the horizontal displacement is three times the maximum height reached by the bullet. The initial velocity of the bullet is 150 m/s.

Calculate the angle of projection. (5)

[17]

QUESTION 2

2.1 Define *angular displacement.* (2)

2.2 The wheel of a motorbike has a diameter of 40 cm and accelerates from 4 rad/s to 10 rad/s in 20 seconds.

Calculate the following:

2.2.1 The angular acceleration of the wheel (2)

2.2.2 The angular displacement of the wheel in radians (2)

2.2.3 The number of revolutions completed by the wheel during this time (2)

2.3 The wheel of a belt drive has a diameter of 36 cm and rotates at 500 r/min. The belt is subjected to an effective force of 400 N. Calculate the power transmitted by the belt. (4)

[12]

QUESTION 3

3.1 Define *Newton's Third Law.* (1)

3.2 A locomotive is pulling a train with a mass of 150 ton up a hill with an incline of 10° at a constant velocity of 72 km/h. The train experiences a frictional force of 8 000 N.

Calculate the power required by the engine of the locomotive to pull the train. (3)

3.3 A toy car with a mass of 1 kg is projected up an incline of 1 : 5 at an initial velocity of 1,5 m/s. Calculate the distance that the car will move up the incline before coming *to* rest. Ignore all losses due to friction. (5)

[9]

QUESTION 4

4.1 A light, horizontal beam ABCDE is 7 metres long. It is supported at two points, 8 and 0, each 1 metre from the ends of the beam. The beam carries the following loads:
A concentrated load of 30 kN at the left end, point A
A concentrated load of 40 kN, 2 metre from the right end at point C
A concentrated load of 40 kN, at the right end, point E
A uniformly distributed load of 5 kN/m over the first 5 metres from the left

4.1.1 Make a neat, labelled diagram of the beam as described above. (1)

4.1.2 Calculate the reactions of the supports at points 8 and D. (3)

4.1.3 Calculate the bending moments at the supports and also at a point 5 metres from the left. (3)

4.1.4 Draw a shear force diagram and a bending moment diagram and show all the main values on the diagrams. (5)

4.1.5 Calculate the magnitude of the maximum bending moment and its position. (4)

4.2 Write down the positions of the following:

4.2.1 The centroid of a triangular plate with a perpendicular height *h,* resting on one of its sides and measured from this baseline. (1)

4.2.2 The centre of gravity of a cone with a perpendicular height *h* and radius *r,* resting on its circular base. (1)

[18]

QUESTION 5

5.1 Define the term *density.* (1)

5.2 Name TVVO important facts relating to the pressure exerted by liquids. (2)

5.3 The information below refers to a single-acting hydraulic press:

Cross-sectional area of the plunger = 30% of that of the ram cross- sectional area
Stroke length of the plunger = 0,2m
Force exerted on the plunger = 600 N
Cross-sectional area of the ram = 0,2 m2

Ignore all losses and calculate:

5.3.1 The volume of liquid displaced after 12 pumping strokes of the plunger (2)

5.3.2 The distance moved by the ram, in mm, after 1 stroke of the plunger (2)

5.3.3 The force exerted by the ram (2)

5.3.4 The mechanical advantage of the press (2)

5.3.5 The pressure in the liquid (2)

5.4 The plungers of a two-cylinder, single-acting pump have diameters of 8 cm each and stroke lengths of 30 cm each. The pressure during the delivery stroke is 1 MPa.

Calculate the following:

5.4.1 The power required to drive the pump at 200 r/min if the efficiency of the motor is 85% (4)

5.4.2 The volume of water delivered per minute if there is no slip (5)

[20]

QUESTION 6

6.1 Explain the difference between *tensile stress* and *compressive* stress. (2)

6.2 A concrete pillar with a diameter of 60 cm is used in a construction. The pillar is subjected to a compressive stress of 5 MPa. Calculate the maximum load allowed on the pillar. (3)

6.3 A bar with a square profile of 25 mm x 25 mm is subjected to a tensile test. A load of 100 kN causes an extension of 0,3 mm. The initial length of the bar was 330 mm.

Calculate the following:

6.3.1 The stress in the bar (2)

6.3.2 The strain (2)

6.3.3 Young's modulus of elasticity of the material (2)

[11]

QUESTION 7

7.1 Define *Boyle's Jaw* using a brief definition, writing down an equation and drawing a graph to illustrate the law. (5)

7.2 A piece of thin solder wire (an alloy of lead) with an original length of 10 cm is used during a demonstration to illustrate the concept of linear expansion. It is established that the change in temperature of the wire is 60 ºC. The final length of the wire is 10,01722 cm. Calculate the linear expansion coefficient of the solder wire. (3)

7.3 Nitrogen gas is contained in a closed cylinder with a volume of 10 l at a temperature of 15 ºC. The pressure inside the cylinder is 1 600 kPa. The temperature decreases to 5 °C.

Calculate:

7.3.1 The pressure at the lower temperature (3)

7.3.2 The mass of the nitrogen gas contained in the cylinder if the gas constant of nitrogen gas is 260 J/kg.K (2)

[13]

TOTAL: 100

ENGINEERING SCIENCE N4

FORMULA SHEET

Any applicable formula may also be used.

$S = \frac{u+v}{2} \times t$	$a = \alpha R$	$H.V. = \frac{F_p}{F_h} = M.A.$
$\overline{V} = \frac{s}{t}$	$v = \pi DN$	$AV = mgh = WD$
$v = u + at$	$T = FR$	$\underline{Q} = mc\Delta t$
$s = ut + \frac{1}{2}as^2$	$AV = T\theta = WD$	$\Delta l = lo\alpha\Delta t$
$v^2 = u^2 + 2as$	$P = 2\pi NT$	$\beta = 2\alpha$
$v_g = \frac{u+v}{2}$	$P = Fv$	$\gamma = 3\alpha$
$\omega = 2\pi N$	$P = T\omega$	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
$\omega = \frac{\theta}{t}$	$F_\alpha = ma$	$PV = mRT$
$\theta = \frac{\omega_2 + \omega_1}{2} \times t$	$E_p = mgh$	$\in = \frac{x}{l}$
$\omega_2 = \omega_1 + \alpha t$	$E_k = \frac{1}{2}\mu\varpi^2$	$E_k = \frac{\sigma}{\in}$
$\theta = \omega_2 t + \frac{1}{2}\alpha t^2$	$P = \frac{F}{A}$	$\sigma = \frac{F}{A}$
$v = \omega R$	$m = \rho \times vol$	$E = \frac{Fl}{Ax}$
$\theta = 2\pi n$	$P = \rho gh$	$\overline{y} = \frac{A_1y_1 \pm A_2y_2 \ldots}{A_1 \pm A_2 \ldots}$
$S = R\theta$	$\frac{W_r}{F_p} = \frac{D^2}{d^2}$	$\overline{y} = \frac{v_1y_1 \pm v_2y_2 \ldots}{v_1 \pm v_2 \ldots}$
$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta}$	$W.D. = P \times V = A..V.$	

INFORMATION SHEET

PHYSICAL CONSTANTS

QUANTITY	CONSTANT
Atmospheric pressure	101,3 kPa
Density of copper	8 900 kg/m^3
Density of aluminium	2 770 kg/m^3
Density of gold	19 000 kg/m^3
Density of alcohol (ethyl)	790 kg/m^3
Density of mercury	13 600 kg/m^3
Density of platinum	21 500 kg/m^3
Density of water	1 000 kg/m^3
Density of mineral oil	920 kg/m^3
Density of air	1,05 kg/m^3
Electrochemical equivalent of silver	1,118 mg/C
Electrochemical equivalent of copper	0,329 mg/C
Gravitational acceleration	9,8 m/s^2
Heat value of coal	30 MJ/kg
Heat value of anthracite	35 MJ/kg
Heat value of petrol	45 MJ/kg
Heat value of hydrogen	140 MJ/kg
Linear coefficient of expansion of copper	17 x 10^{-6}/°C
Linear coefficient of expansion of aliminium	23 x 10^{-6}/°C
Linear coefficient of expansion of steel	12 x 10^{-6}/°C
Linear coefficient of expansion of lead	54 x 10^{-6}/°C
Specific heat capacity of steam	2 100 J/kg.°C
Specific heat capacity of water	4 187 J/kg.°C
Specific heat capacity of aluminium	900 J/kg.°C
Specific heat capacity of oil	2 000 J/kg.°C
Specific heat capacity of steel	500 J/kg.°C
Specific heat capacity of copper	390 J/kg.°C

Marking Guidelines

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

AUGUST 2012

NATIONAL CERTIFICATE

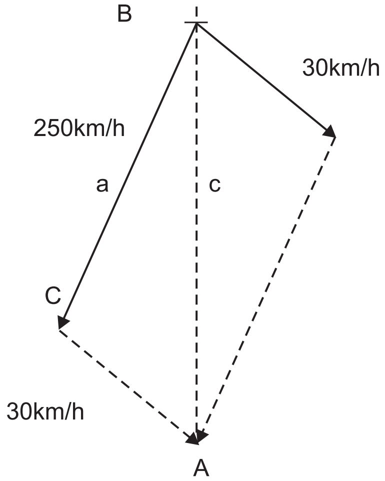
ENGINEERING SCIENCE N4

(15070434)

This marking guidelines consists of 10 pages.

QUESTION 1

1.1



$$\frac{\sin b}{b} = \frac{\sin A}{a}$$

$\sin B = 30\sin 45°/250$

$B = 4{,}868°$

$C = 180° - 45° + 4{,}868°)$

$= 130{,}13°$

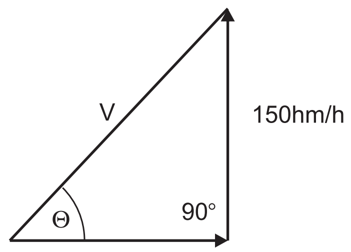
$c^2 = a^2 + b^2 - 2ab\cos C$

$= 250^2 + 30^2 - 2(250)(30)\cos 130{,}13°$

$C = 270{,}31$ km/h

The pilot must steer South 4,868° West OR West 85,132° South (4)

1.2



$V^2 = 150^2 + 100^2$

$V = 180{,}28$ km/h

$\Theta = \tan^{-1} 150/100$

$= 56{,}31°$

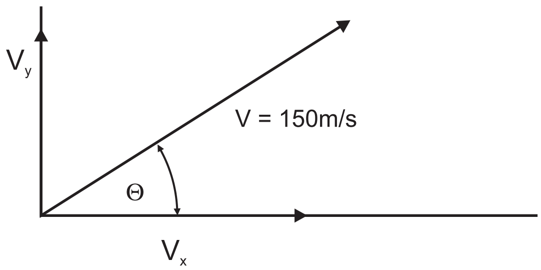
Direction of aeroplane is East 56,31º North OR North 33,69° East (4)

1.3.1 $v = u + gt$

$0 = 50 - 9{,}8 \times t$
$t = 5{,}1$ sec (upwards)
total $t = t(up) + t(down)$ (2)
$= 10{,}2$ sec

1.3.2 $s = ut + 1/2gt^2$
$= 50(5{,}1) + ½ (-9{,}8)(5{,}1)^2$
$= 255 - 127{,}45$
$= 127{,}55$ meter (2)

1.4



Resolve the velocity into the vertical and horizontal components:
$V_x = 150 cos\Theta$
$V_y = 150 sin\Theta$

Time for vertical displacement:

$v = u + gt$
therefore $t = (v - u) + g$
$= (0 - 150\ sin\Theta) \div g$
$= 15{,}306\ sin\Theta$ seconds

s(height) $= (v^2 - u^2) \div 2g$
$= 0 - (150 \times sin\Theta)^2 \div 2(-9{,}8)$
$= 1147{,}96\ (sin\Theta)^2$ meter

s(horizontal) $= 3 \times V_Y$
$= 3 \text{ X } 1147{,}96\ (sin\Theta)^2$
But $s = V_X \times t$

$150 \times cos\Theta \times 2 \times 15{,}306\ sin\Theta$

Therefore $= 3 \times 1147{,}96\ (sin\Theta)^2 = 150 \times cos\Theta \times 2 \times 15{,}306\ sin\Theta$ (5)
Simplify: $sin\Theta/cos\Theta = 4591{,}8/3443{,}88$
$tan\Theta = 1{,}333$
$\Theta = 53{,}13°$

[17]

QUESTION 2

2.1 Angular displacement is the distance travelled on a circular route, measured in radians (2)

2.2.1 $v = u + at$
therefore $\omega_2 = \omega_1 + \alpha t$
$\alpha = (\omega_2 - \omega_1)/t$
$= (10 - 4)/20$
$= 0{,}3$ rad/s^2 (2)

2.2.2 $\theta = (\omega_2)^2 - (\omega_1)^2 \alpha$
$= (100 - 16)/ (2 \times 0{,}3)$
$= 140$ radians (2)

2.2.3 8 = 2rrn = (Wz- W1)/ a
n = 8/2rr
= 140/2rr
= 22,28 rev (2)

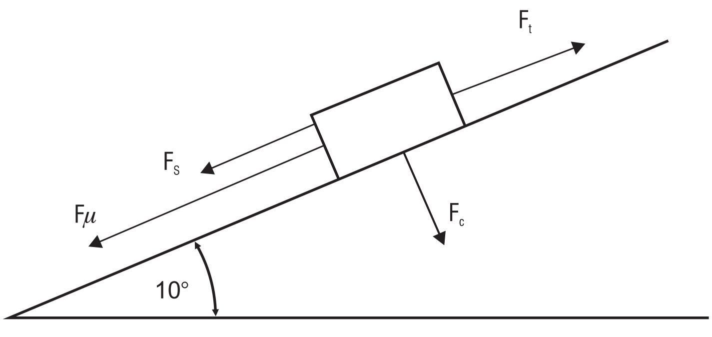
2.3 $P = F_e \times V$
$= F_e \times \omega r$
$= 400 \times (500 \times 2\pi)/60 \times 0{,}18$
$= 3769{,}92$ Watt (4)

[12]

QUESTION 3

3.1 Whenever an object exerts a force on another object, the second object will exert an equal, but opposite force on the first object. ie: For every action there is an equal and opposite reaction force. (1)

3.2

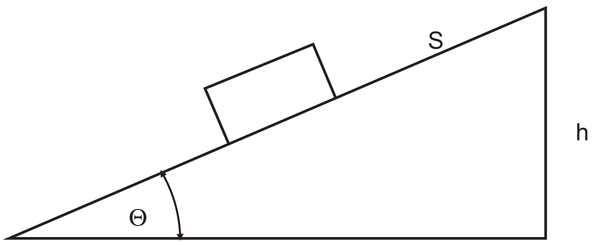


$F_s = mg \times \sin\Theta$
$= 9{,}8 \times 150\ 000 \times \sin 10°$
$= 255\ 262{,}82$N

$P = F_t \times v$
$= (F_R + F_S) \times v$
$= (8000 + 255\ 262{,}82) \times 20$
$= 5\ 265\ 256{,}42$ Watt
$= 5{,}265$ MW

(3)

3.3



Kinetic energy at the bottom = $\frac{1}{2} mv^2$

$= \frac{1}{2} \times 1 \times 1,5^2$

$= 1,125$ J

Loss in kinetic energy is equal to gain in kinetic energy:

Potential energy = m x g x h

$h = E_K/mxg$

$= 1,125/1 \times 9,8$

$= 0,1148$m

Distance up the incline: h/s = sinΘ

$s = h/\sin\Theta$

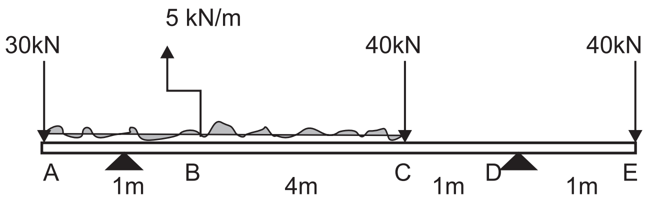
$= 0,1148/11,31^\circ$

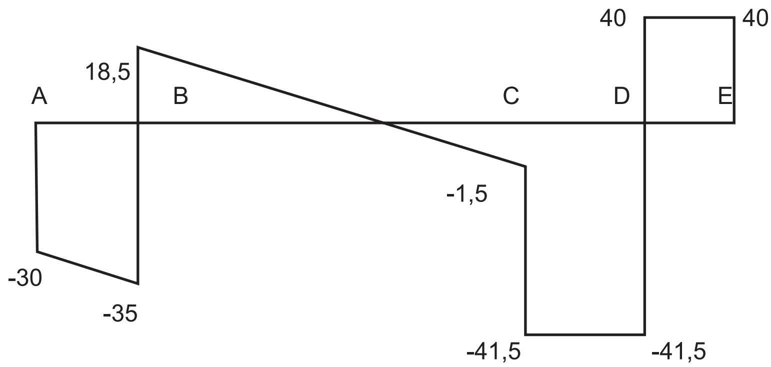
$= 0,585$ metre (5)

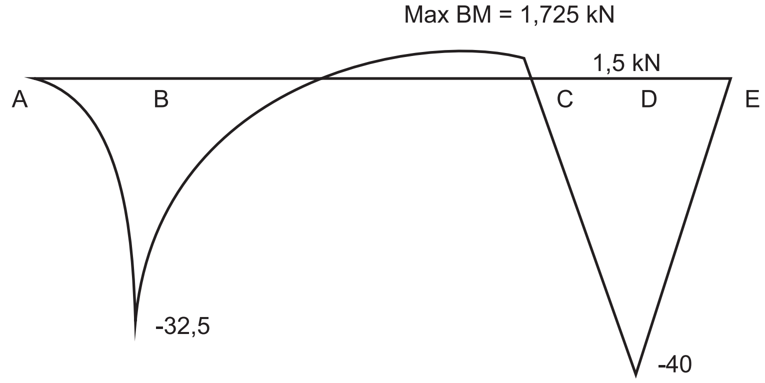
[9]

QUESTION 4

4.1.1 (1)







4.1.2 Take moments about B: (3)
Σ CWM = Σ ACWM
(40 x 4) + (40 x 6) + (25 x 1,5) = (30 x 1) + 5 x D
C = 81,5 kN

Take moments about D:
Σ ACWM = Σ CWM
(40 x 1) + 8 x 5 = (30 x 6) + (25 x 3,5) + (40 x 1)
B = 53,5 kN

Test: Σ↑ F = 81,5 + 53,5 = 135 kN
And Σ↓ F = 30 + 25 + 40 + 40 = 135 kN

4.1.3 BM at A = O (3)
BM at B = -(20 x 23) - (40 x 4) - (40 x 6) + (81 ,5 x 5) = - 32,5 kNm
BM at C = -(40 x 2) + (81,5 x 1) = 1,5 kNm
BM at 0 = - 40 x 1 = - 40 kNm
BM at E= 0

4.1.4 See attached (5)

4.1.5 Let the maximum bending moment be at z (4)

Shear forces at z = 0

Therefore - 30 + 53,5 - 5 x x = 0

x = 4,7 meter

therefore maximum bending moment is 4,7 meter from the left

Maximum bending moment at z:- (30 x 4,7) + (53,5 x 3,7) - (5 x 4,7 x 2,35)
= 1,725 kNm

4.2.1 $= 1/3 \times h$ (1)

4.2.2 $G: \bar{x} = r \; and \; \bar{y} = \frac{1}{4} \times h$ (1)

[18]

QUESTION 5

5.1 The density of a substance is the mass per unit volume of the substance and is measured in kg/m^3 (1)

5.2 Pressure has the same magnitude in all directions at any point in a liquid. (2)
The pressure in a liquid is the same at all points on the same horizontal plane in its container.
The pressure in a liquid is independent of the size and shape of its container.
Pressure in a liquid increases with depth, and is directly proportional to the depth.
Pressure exerted by a fluid depends on the density of the fluid, the greater the density the greater the pressure exerted.
When pressure is exerted on the surface of the fluid, then the pressure is propagated at the same magnitude in all directions by the liquid. (any two)

5.3.1 Cross sectional area of plunger = 30/100 x 0,2 = 0,06 m^3 (2)
Volume = a x h x n
= 0,06 x 0,2 x 12
= 0,144 m^3

5.3.2 A x H = n x a x h (2)
0,06 x 0,2 = 0,2 x H
H = 0,06 m
H = 60 mm

5.3.3 F/A = f/a (2)

F = (0,2 X 600)/0,06
F = 2 000 N

5.3.4 MA = F/f (2)
= 2000/600
= 3,333

5.3.5 Pressure = f/a (2)
= 600/0,06
= 10 kPa
Or
Pressure = F/ A
= 2000/0,2
= 10 kPa

5.4.1 V = πd2/4 x h x 2 (4)
= π x (0,08)2/4 x 0,3 x 2
= 0,00302 m^3

Power = Pressure x Volume
= 1 x 10^6 x 0,00302 x 200/60 x 100/85
= 11 843,14 Watt
= 11,843 kW

5.4.2 Volume = $\pi d^2/4$ x h x speed of pump (5)
= 0,00302 x 200
= 0,604 m^3

[20]

QUESTION 6

6.1 Tensile stress is produced when an object is subjected to a tensile load and it may cause the object to elongate and with a reduction in cross-sectional area. (2)
Compressive stress is produced when an object is subjected to a compressive load and it may cause the object to shorten and with an increase in cross-sectional area.

6.2 P = F/A (3)
F = P x A
= 5 X 106 x $\pi(0,6)^2/4$
= 1,414 MN

6.3.1 $\sigma = F/a$ (2)
= 100 x 10^3 /$0,025^2$
= 160 MPa

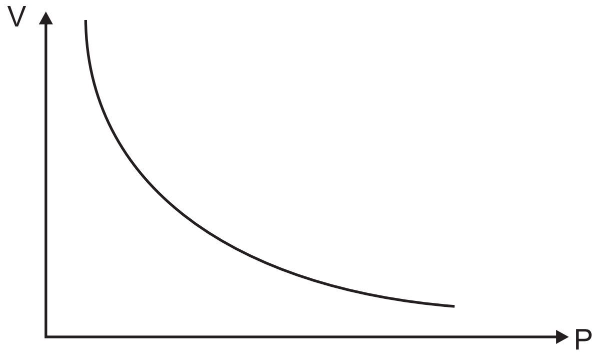
6.3.2 $\in = x/l = 0,3 \times 10^{-3}/330 \times 10^{-3} = 0,909 \times 10^{-3}$ (2)

6.3.3 $E = \sigma/\in = 160 \times 10^6 /0,909\, x\, 10^{-3} = 176\ GPa$ (2)

[11]

QUESTION 7

7.1 The volume of a given mass of gas is inversely proportional to the pressure exerted on it, provided that the temperature is kept constant. Thus for a given mass of gas P x V =constant. Therefore $P_1V_1 = P_2V_2$ (5)



7.2 $\Delta l = l_o \times \alpha \times \Delta t$ (3)
1,722 x 10^{-4} = 0,1 x α x 60
α = 2,87 x 10^{-5} /ºC

7.3 T_1 = 288 K
T_2 = 278 K

7.3.1 $P_1V_1/T_1 = P_2V_2/T_2$ (3)
1600 X 10/288 = P_2 X 10/278
P_2 = 1544,44 kPa

7.3.2 PV = mRT (2)
m = PV/RT
= (1600 X 10^3 X (10 X 10^{-3}) / (260 X 288)
= 0,214 kg

[13]

TOTAL: 100

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